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Name:

Analysis I — Quiz 5

2.11.10

Q5.1. Use part (a) to proof part (b) and (c) in the inheritance principle.

INHERITANCE PRINCIPLE. Let (X, d_X) be a metric space and $A \subseteq X$. Then (A, d_A) becomes a metric space when setting $d_A = d_X|_{A \times A}$, that is, $d_A(a, b) = d_X(a, b)$ for $a, b \in A$. Further, the following hold:

(a) $B \subset A$ is open in (A, d_A) if and only there exists $\widetilde{B}$ open in (X, d_X) such that $B = A \cap \widetilde{B}$.

(b) $B \subset A$ is closed in (A, d_A) if and only there exists $\widetilde{B}$ closed in (X, d_X) such that $B = A \cap \widetilde{B}$.

(c) $B \subset A$ is clopen (closed and open) in (A, d_A) **if** there exists $\widetilde{B}$ clopen in (X, d_X) such that $B = A \cap \widetilde{B}$.

Q5.2. Consider the metric space $\mathbb{R}$. Find $A \subseteq \mathbb{R}$ and $B \subseteq A$ is clopen (closed and open) in A equipped with the standard metric of $\mathbb{R}$ so that there exists no clopen $\widetilde{B} \subseteq \mathbb{R}$ such that $B = A \cap \widetilde{B}$. (No need for a proof, just provide a valid example.)