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# Analysis II — Problem Set 10

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**10.1.** Let $v_i = (a_{i1}, a_{i2}, \dots, a_{in})$, $i = 1, \dots, n$ be vectors in $\mathbb{R}^n$. Prove that the $n$-parallelogram $P(v_1, v_2, \dots, v_n)$, spanned by these vectors, i.e., the set

$$P(v_1, v_2, \dots, v_n) = \{\alpha_1 \cdot v_1 + \alpha_2 \cdot v_2 + \cdots + \alpha_n \cdot v_n \; \alpha_i \in [0,1]\}$$

has volume

$$\det(a_{ij})_{i,j=1,\dots,n}$$

**10.2.** Find the dimensions of the rectangular box in the 3-dimensional space of maximum volume given that the surface area is 10 m$^2$.

**10.3.** The region $\mathcal{D}$ is bounded by the segments $x = 0$, $0 \le y \le 1$; $y = 0$, $0 \le x \le 1$; $y = 1$, $0 \le x \le \frac{3}{4}$ and by an arc of the parabola $y^2 = 4(1-x)$. Consider a mapping into the $(x, y)$ from $(u, v)$ plane defined by the transformation $x = u^2 - v^2$, $y = 2uv$. Sketch $\mathcal{D}$ and also the two regions in the $(u, v)$ plane which are mapped into it. Evaluate

$$\int_{\mathcal{D}} \frac{dxdy}{(x^2 + y^2)^{1/2}}$$

**10.4.** Compute the integral $\int\int_V\int \frac{dxdydz}{(1+x+y+z)^3}$ where $V$ is the region bounded by the surfaces $x + y + z = 1$, $x = 0$, $y = 0$ and $z = 0$.

**10.5.** Compute the following integrals:

(a)
$$\int\int_{\mathcal{S}} \frac{x^2}{x^2 + 4y^2} dxdy,$$
where $\mathcal{S}$ is the region between the two ellipses $x^2 + 4y^2 = 1$, $x^2 + 4y^2 = 4$.
*Hint*: use the substitution $x = \rho\cos(\theta)$, $y = \frac{1}{2}\rho\sin(\theta)$.

(b)
$$\int\int_{\mathcal{D}} \frac{2x - y}{(x + y)^2} dxdy,$$
where $\mathcal{D}$ is bounded by $x + y = 1$, $x + y = 2$, $2x - y = 1$, $2x - y = 3$.

**10.6.** **Bonus problem.** Write the iterated integral

$$\int_0^1 \left( \int_y^{y^{1/3}} e^{-x^2} dx \right) dy$$

as an integral over a subset of the plane and compute it.