Jacobs University Bremen
School of Engineering and Science
Götz Pfander, Sergei Markouski,
Alex Sava

Spring Term 2008

Analysis II — Problem Set 3

Issued: 20.02.08 Due: 27.02.08, noon

3.1. Compute the following indefinite integrals

(a) $\int \arcsin x \, dx$,

(b) $\displaystyle\int \frac{8x+1}{x^2+x+2}\, dx$,

(c) $\displaystyle\int \frac{x^3 \mathrm{arcctg} x}{1+x^2}\, dx$.

3.2. Prove that $\displaystyle\lim_{n\longrightarrow\infty} \int_n^{n+p} \frac{\sin x}{x} dx = 0$, where $p > 0$.

3.3. Compute the following improper integrals

(a) $\displaystyle\int_0^{+\infty} \frac{x^2+1}{x^4+1} dx$,

(b) $\displaystyle\int_0^1 \frac{dx}{(2-x)\sqrt{1-x}}$,

(c) $\displaystyle\int_{-\infty}^{\infty} \frac{dx}{(x^2+x+1)^2}$.

3.4. Let φ be monotonically increasing continuously differentiable function which maps $[0, +\infty)$ onto $[a, b)$. Prove that for any continuous on $[a, b]$ function f the following equality holds:

$$\int_a^b f(x)dx = \int_0^\infty f(\varphi(t))\varphi'(t)dt$$

3.5. Check whether the series

$$\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+x} \right)$$

converges for $x \in \mathbb{R} - \{-1, -2, -3, ...\}$.

3.6. Let f be 1–periodic and satisfy $f(x) = -f(-x)$ for all x and $f(x) = x(\frac{1}{2} - x)$ for $0 \leq x \leq \frac{1}{2}$. Draw f using MatLab (or a similar computer package). Assume that $f(x) = \sum_{n=-\infty}^{\infty} \widehat{f}(n)\, e^{2\pi i x n}$ with pointwise convergence for all x. Calculate the Fourier coefficients $\widehat{f}(n)$, $n \in \mathbb{Z}$, simplify $\sum_{n=-\infty}^{\infty} \widehat{f}(n)\, e^{2\pi i x n}$, and conclude that $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^3} = \frac{\pi^3}{32}$. Also, draw $\sum_{n=-N}^{N} \widehat{f}(n)\, e^{2\pi i x n}$ for $N = 0, 1, 3, 7$.