Jacobs University
School of Engineering and Science
Götz Pfander

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Functional Analysis — Problem Set 7

Issued: 08.05.10 Due: 17.5.10, hand over to TA

7.1. Conway, page 176, exercise 1.

7.2. Conway, page 176, exercise 3.

7.3. Conway, page 177, exercise 6.

7.4. Conway, page 177, exercise 15.

7.5. Conway, page 177, exercise 16 (Bonus).

7.6. What is σ_p, σ_c, σ_r of the shift operator $S : l^1(\mathbb{N}) \longrightarrow l^1(\mathbb{N})$, $(s_1, s_2, s_3, \ldots) \mapsto (s_2, s_3, s_4, \ldots)$. Also find the spectrum of S^*. Moreover, what changes if we consider instead $S : l^2(\mathbb{N}) \longrightarrow l^2(\mathbb{N})$, $(s_1, s_2, s_3, \ldots) \mapsto (s_2, s_3, s_4, \ldots)$.

7.7. For X Banach, $T \in B(X)$, $\lambda, \mu \in \rho(T)$show $R_\lambda(T) - R_\mu(T) = (\mu - \lambda)R_\lambda(T)R_\mu(T)$.

7.8. For K compact in $\mathbb{C}$, construct $T \in B(l^2(\mathbb{N}))$ with $\sigma(T) = K$. Hint: consider multiplication operators. (Bonus)

7.9. For X Banach show that $\{T \in B(X) : T \text{ invertible } \}$ is open in $B(X)$. Moreover, $A : \{T \in B(X) : T \text{ invertible } \} \longrightarrow \{T \in B(X) : T \text{ invertible } \}$, $T \mapsto T^{-1}$ is bounded. Hint: use Neumann series.