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Local positivity of principally polarized abelian threefolds

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Introduction

In recent years there has been considerable interest in understanding the local positivity of ample line bundles on algebraic varieties. Seshadri constants, introduced by Demailly [4], emerged as a natural measure of the local positivity of a line bundle. These invariants are very hard to control and their exact value is known only in very few cases.

Let X be a smooth projective variety of dimension n, and let L an ample line bundle on X. Then the real number

$$\varepsilon(L,x) =_{\text{def}} \inf_{C \ni x} \frac{L \cdot C}{\operatorname{mult}_x C}$$

is the *Seshadri constant of* L *at* $x \in X$. (The infimum is taken over all irreducible curves passing through x.) One has the universal upper bound $\varepsilon(L,x) \le \sqrt[n]{L^n}$, which follows easily from Kleiman's ampleness criterion. If X is an abelian variety, then $\varepsilon(L) = \varepsilon(L,x)$ does not depend on the point $x \in X$, and one has the lower bound $\varepsilon(L) \ge 1$. Nakamaye [8] showed that the equality $\varepsilon(L) = 1$ has strong geometric consequences: X splits in this case as a product of polarized varieties $X' \times E$, where E is an elliptic curve and X' is a lower-dimensional abelian variety. If $(X,L) = (JC, \Theta_C)$ is the Jacobian of a smooth curve C of genus g, then $\varepsilon(\Theta_C) \le \sqrt{g}$ by a result of Lazarsfeld [6]. For abelian surfaces of Picard number one, a complete picture has recently been given in [2]. For higher-dimensional abelian varieties, however, no explicit values of Seshadri constants are known up to now.

As a first step towards understanding the higher-dimensional picture, we determine in the present paper the Seshadri constants of principally polarized abelian threefolds. A principally polarized abelian threefold is either a product or a Jacobian of a smooth curve of genus 3 (see [5]). Our result shows that Seshadri constants reflect moduli properties of abelian varieties. We prove:

Theorem 1 *Let* (X,L) *be a principally polarized abelian threefold. Then*

$$\varepsilon(X,L) = \begin{cases} 1 & , \textit{if } (X,L) \textit{ is a polarized product} \\ \frac{3}{2} & , \textit{if } (X,L) = (JC,\Theta_C) \textit{ is the Jacobian} \\ & \quad \textit{of a hyperelliptic curve } C \\ \frac{12}{7} & , \textit{otherwise.} \end{cases}$$

Note that the inequality $\varepsilon(JC,\Theta_C)\le\frac{3}{2}$ for hyperelliptic C follows from [6, Proposition (ii)].

The theorem has two immediate consequences that we find interesting. First, it shows that the Seshadri constant of a principally polarized abelian threefold is always a rational number. And secondly, it provides an upper bound on minimal period lengths. Writing X as a quotient $\mathbb{C}^n/\Lambda$ by a lattice Λ, we can view the first Chern class of L as a positive definite Hermitian form H on $\mathbb{C}^n$. The *minimal period length* of (X,L) is then the real number

$$m(X,L) =_{\rm def} \min_{\substack{\lambda\in\Lambda\\ \lambda\neq 0}} H(\lambda,\lambda) \ ,$$

i.e., the square of the length (with respect to H) of the shortest period. As as consequence of Lazarsfeld's inequality [6], Theorem 1 then implies:

Corollary 2 *Let (X,Θ_C) be a non-hyperelliptic three-dimensional Jacobian. Then*

$$m(X,\Theta_C)\le\frac{48}{7\pi} \ .$$

It is natural to wonder what a higher-dimensional analogue of Theorem 1 might look like. For instance, one may ask which properties of a curve C might influence the value of the Seshadri constant $\varepsilon(JC,\Theta_C)$. Our geometric idea that underlies the proof of Theorem 1, however, does not seem to have an obvious analogue in dimension bigger than three.

1. Submaximal subvarieties

As in the introduction, consider a smooth projective variety X and an ample line bundle L on X. We begin by recalling that the definition of the Seshadri constant $\varepsilon(L,x)$ at some point $x\in X$ can be reformulated as

$$\varepsilon(L,x)=\sup\{\,\varepsilon\in\mathbb{R}\mid f^*L-\varepsilon E \text{ is nef}\,\} \ , \qquad (1)$$

where $f:Y\to X$ is the blow-up of X at x with exceptional divisor E over x.

An effective cycle Z on X of dimension d, with $1\le d<n$, will be called *L-submaximal* at a point $x\in X$, if

$$\sqrt[d]{\frac{L^d\cdot Z}{\operatorname{mult}_x Z}}<\sqrt[n]{L^n} \ .$$

This terminology is motivated by the fact that by [4, (6.7)] the existence of an L-submaximal cycle at x causes the Seshadri constant $\varepsilon(L,x)$ to be below its maximal possible value $\sqrt[n]{L^n}$. In fact, in this case

$$\varepsilon(L,x)\le\sqrt[d]{\frac{L^d\cdot Z}{\operatorname{mult}_x Z}} \ . \qquad (2)$$

We will say that Z *computes the Seshadri constant of L at x*, if equality holds in (2). The real-valued Nakai-Moishezon criterion [3] implies that for any point $x\in X$ either

- $\varepsilon(L,x)$ is maximal, i.e., $\sqrt[n]{L^n}$, or

or

- there is an effective cycle Z on X – in fact even an irreducible subvariety – computing $\varepsilon(L,x)$

(cf. [10, Proof of Proposition 4]). The dimension of the subvariety Z in question is, however, unknown a priori.

For the 3-dimensional case, the following proposition gives restrictions on the subvarieties computing $\varepsilon(L,x)$, provided that there is a submaximal divisor in a multiple of L.

Proposition 1.1 *Let X be a smooth projective variety of dimension three, let $x \in X$ be a point and L an ample line bundle on X. Suppose that for some integer $p > 0$ there is a divisor $D \in |pL|$ that is L-submaximal at x. Then:*

(a) *Every L-submaximal curve on X lies on the support of D.*

(b) *The Seshadri constant $\varepsilon(L,x)$ is computed either by a curve lying on the support of D or by a divisor D_0. If the Néron-Severi group of X is of rank one, then D_0 is an irreducible component of D.*

Proof. For the proof of (a) note to begin with that from the submaximality assumption on D one has

$$\operatorname{mult}_x D > p\sqrt[3]{L^3} .$$

Suppose then to the contrary that there is an L-submaximal curve C that is not contained in $\operatorname{supp} D$. Thus C and D meet properly, so that

$$pL \cdot C = D \cdot C \geq \operatorname{mult}_x D \cdot \operatorname{mult}_x C .$$

Using the submaximality of C we get

$$\operatorname{mult}_x D < \frac{pL \cdot C}{\operatorname{mult}_x C} < p\sqrt[3]{L^3} ,$$

and this contradiction proves (a).

Turning to (b), we have $\varepsilon(L,x) < \sqrt[3]{L^3}$ from the assumption on D. Therefore, if there is a curve $C \subset X$ computing $\varepsilon(L,x)$, the above claim shows that C is contained in $\operatorname{supp} D$. So in this case statement (b) holds and we are done. If $\varepsilon(L,x)$ is not computed by a curve, then there exists a sequence (C_n) of distinct irreducible curves $C_n \subset X$ with the property that

$$\lim_{n\to\infty} \frac{L \cdot C_n}{\operatorname{mult}_x C_n} = \varepsilon(L,x) .$$

We may assume – again because of $\varepsilon(L,x) < \sqrt[3]{L^3}$ – that all the curves C_n are L-submaximal at x, and hence that

$$C_n \subset \operatorname{supp} D \qquad \text{for all } n$$

according to (a). If there is no curve computing $\varepsilon(L,x)$, then the real-valued Nakai-Moishezon criterion implies that there is an irreducible (and, of course, L-submaximal) divisor D_0 on X computing $\varepsilon(L,x)$.

Under our additional assumption on the Néron-Severi group, the submaximal divisor D_0 is algebraically proportional to D. The arguments in the proof of (a) above still hold when we replace D by D_0, so that we conclude that

$$C_n \subset \operatorname{supp} D_0 \cap \operatorname{supp} D \qquad \text{for all } n \ .$$

But then D_0 must be a component of D, and this implies the second part of statement (b). □

Note that if D is irreducible and $\sqrt{\frac{L^2\cdot D}{\operatorname{mult}_x D}}$ is an irrational number, then exactly one of the cases in statement (b) happens.

The proof of the main theorem relies on the following criterion, which will allow us to construct curves computing Seshadri constants on 3-dimensional Jacobians. We start by fixing some notation. Consider a smooth projective curve C. We will denote by $s : C\times C \to JC$ the subtraction map $(x,y)\mapsto \mathcal{O}_C(x-y)$. Further, the numerical equivalence classes of the fibers of the projections $C\times C\to C$ will be denoted by F_1 and F_2, and the diagonal in $C\times C$ by Δ.

Proposition 1.2 *Let C be a smooth projective curve of genus 3, and let $G\subset C\times C$ be an irreducible curve that is numerically equivalent to*

$$a_1F_1 + a_2F_2 - b\Delta \ ,$$

where a_1, a_2 and b are positive integers. If

$$\frac{s^*\Theta\cdot G}{G\cdot\Delta} < \sqrt{3} \ ,$$

then the image curve $s_*G\subset JC$ computes $\varepsilon(\Theta)$, and we have

$$\varepsilon(\Theta) = \frac{s^*\Theta\cdot G}{G\cdot\Delta} \ .$$

Proof. Consider the commutative diagram

$$\begin{array}{ccccc}
\Delta & = & \Delta & \longrightarrow & E \\
\downarrow & & \downarrow & & \downarrow \\
C\times C & \xrightarrow{\widetilde{s}} & \widetilde{\Sigma} & \hookrightarrow & \mathrm{Bl}_0(JC) \\
\| & & \downarrow & & \downarrow f \\
C\times C & \xrightarrow{s} & \Sigma & \hookrightarrow & JC
\end{array}$$

where Σ denotes the image of s in JC and f is the blowing-up of JC at 0 with exceptional divisor E. We claim that

$$\text{for } \varepsilon \le \sqrt[3]{\Theta^3}, \text{ the } \mathbb{R}\text{-divisor } f^*\Theta - \varepsilon E \text{ is nef if and only if } s^*\Theta - \varepsilon\Delta \text{ is.} \tag{3}$$

The divisor $s^*\Theta - \varepsilon\Delta$ is the pullback of $f^*\Theta - \varepsilon E$ under $\widetilde{s}$, hence the 'only if' part is clear. Turning to the 'if' part, let us assume that there is a curve $R \subset \mathrm{Bl}_0(JC)$ such that

$$(f^*\Theta - \varepsilon E)R < 0 \ .$$

The surface Σ is Θ-submaximal at the origin (see [6, Proof of Proposition]), hence by Proposition 1.1 the image curve f_*R lies on Σ, so R lies on $\widetilde{\Sigma}$. Therefore

$$(s^*\Theta - \varepsilon\Delta)\widetilde{s}^*R = (f^*\Theta - \varepsilon\Delta)R < 0 \ ,$$

and this proves (3).

Denote by ι the involution on $C \times C$ interchanging the factors, and consider the curve

$$H =_{\mathrm{def}} G + \iota^*G \ .$$

It is symmetric with respect to ι and satisfies the numerical equivalence

$$H \equiv (a_1 + a_2)(F_1 + F_2) - 2b\Delta \ .$$

Let

$$\varepsilon_0 =_{\mathrm{def}} \frac{s^*\Theta \cdot G}{G \cdot \Delta} = \frac{s^*\Theta \cdot H}{H \cdot \Delta} = \frac{3(a_1 + a_2)}{a_1 + a_2 + 4b} \ .$$

Then, since $s^*\Theta \equiv 2(F_1 + F_2) + \Delta$ (cf. [9, Theorem 4.2]), we have

$$\begin{aligned} s^*\Theta - \varepsilon_0\Delta &= 2(F_1 + F_2) + (1 - \varepsilon_0)\Delta \\ &= \frac{2}{a_1 + a_2}H + \frac{4b + (1 - \varepsilon_0)(a_1 + a_2)}{a_1 + a_2}\Delta \end{aligned}$$

The point is now that our assumption $\varepsilon_0 < \sqrt{3}$ guarantees that the coefficients at H and Δ are both positive. So, in order to check that $s^*\Theta - \varepsilon_0\Delta$ is nef, it is sufficient to look at its intersection with the irreducible curves G, ι^*G and Δ. We have

$$(s^*\Theta - \varepsilon_0\Delta)\Delta = 4\varepsilon_0 > 0$$

and

$$(s^*\Theta - \varepsilon_0\Delta)G = (s^*\Theta - \varepsilon_0\Delta)\iota^*G = 0 \ ,$$

so that we can conclude that $s^*\Theta - \varepsilon_0\Delta$ is nef, which upon using (1) and (3) shows that $\varepsilon(\Theta) \ge \varepsilon_0$. On the other hand we have for $\delta \ge 0$

$$\begin{aligned} (s^*\Theta - (\varepsilon_0 + \delta)\Delta)H &= -\delta\Delta \cdot H \\ &= -2\delta(a_1 + a_2 + 4b) \le 0 \ , \end{aligned}$$

so that we also get the converse inequality $\varepsilon(\Theta) \le \varepsilon_0$. This completes the proof of the proposition. □

2. Seshadri constants of Jacobians

In this section we give the proof of Theorem 1. We start with some remarks concerning symmetric products of curves. Let C be a smooth projective curve. We denote the d-th symmetric product of C by $C^{(d)}$, and we write its elements as formal sums $p_1 + \ldots + p_d$, where $p_1, \ldots, p_d$ are points on C. Let $p_0 \in C$ be a fixed point. Then there are for every positive integer d two natural maps

$$\begin{aligned} u_d &: C^{(d)} \to JC, \ (p_1 + \ldots + p_d) \mapsto \mathcal{O}_C(p_1 + \ldots + p_d - dp_0) \\ j_{d-1} &: C^{(d-1)} \to C^{(d)}, \ (p_1 + \ldots + p_{d-1}) \mapsto (p_1 + \ldots + p_{d-1} + p_0) \ . \end{aligned}$$

We will use the divisors $\theta_d = u_d^*\Theta$ and $\xi_d = j_{d-1}(C^{(d-1)})$. To alleviate notation, we will omit the indices when they are clear from the context. For a partition $d = n_1 d_1 + \ldots + n_r d_r$ by pairwise distinct numbers $d_1, \ldots, d_r$, we denote by $\delta(d_1^{n_1}, \ldots, d_r^{n_r})$ the associated diagonal in $C^{(d)}$, i.e., the locus of points of the form

$$(\underbrace{p_1 + \ldots + p_1}_{d_1} + \ldots + \underbrace{p_{n_1} + \ldots + p_{n_1}}_{d_1}) + \ldots +$$

$$(\underbrace{p_{n_1+\ldots+n_{r-1}+1} + \ldots + p_{n_1+\ldots+n_{r-1}+1}}_{d_r} + \ldots + \underbrace{p_{n_1+\ldots+n_r} + \ldots + p_{n_1+\ldots+n_r}}_{d_r}).$$

For a detailed exposition of symmetric products of curves, in particular for the computation of the homology classes of diagonals, we refer to [7].

Proof of Theorem 1. If (X, Θ) is a polarized product, then there is an elliptic curve of Θ-degree one on X, and hence the assertion that $\varepsilon(X, \Theta) = 1$ is clear. So we can assume that $(X, \Theta) = (JC, \Theta_C)$ is the Jacobian of a smooth curve C of genus 3.

Suppose first that C is hyperelliptic. Let $\pi : C \to \mathbb{P}^1$ be the 2:1 map, and let $G \subset C \times C$ be the graph of the hyperelliptic involution. Then

$$\begin{aligned} G + \Delta &= \{ (x, y) \in C \times C \mid \pi(x) = \pi(y) \} \\ &= (\pi \times \pi)^*(\Delta_{\mathbb{P}^1 \times \mathbb{P}^1}) \\ &\in |p_1^*\pi^*\mathcal{O}_{\mathbb{P}^1}(1) + p_2^*\pi^*\mathcal{O}_{\mathbb{P}^1}(1)| \ , \end{aligned}$$

where p_1 and p_2 are the projections $C \times C \to C$. So

$$G \equiv 2(F_1 + F_2) - \Delta \ ,$$

and one checks right away that G satisfies the hypotheses of Proposition 1.2, and therefore

$$\varepsilon(\Theta) = \frac{3}{2}$$

in this case.

Suppose now that C is non-hyperelliptic. Geometrically, the idea of the proof is as follows. Under the canonical embedding $C \hookrightarrow \mathbb{P}^2$, we may view C as a smooth plane quartic curve. The tangent line at a point $p \in C$ cuts out a divisor $x + y + 2p$ on C. Moving the point p, we get a 1-dimensional family G of points $(x, y) \cup (y, x)$ on $C \times C$. By taking the image of G in JC under the subtraction map, we will get a curve that computes $\varepsilon(JC, \Theta)$.

Turning to the details, we consider the canonical variety

$$C_4^2 = \{D \in C^{(4)} : \dim |D| \geq 2\} \subset C^{(4)}$$

and its pullback W to C^4 via the quotient map $\pi_4 : C^4 \to C^{(4)}$. In C^4 we will need the diagonals $\Delta_{ij} = \{p_i = p_j\}$ and $\Delta_{ijk} = \{p_i = p_j = p_k\}$ as well as the fibers f_i of the projections to C. As shown in the following diagram, we consider the intersection $V = W \cap \Delta_{34}$, its projection G to C^2, and finally the image Γ of G in JC under the subtraction map s.

$$\begin{array}{ccc} C^4 & \xrightarrow{\pi_4} & C^{(4)} \\ \cup\uparrow & & \uparrow\cup \\ W & \longrightarrow & C_4^2 \\ \cup\uparrow & & \\ V = W \cap \Delta_{34} = \{(x, y, z, z) \in C^4 \ : \ x + y + 2z \in |K_C|\} & & \end{array}$$

$$\begin{array}{ccc} & V & \\ {}^{pr_{12}}\swarrow & & \searrow^{pr_{34}} \\ C^2 \supset G & & \Delta \subset C^2 \\ \downarrow s & & \\ \Gamma \subset JC & & \end{array}$$

By [1, VII.5], one has

$$[C_4^2] = \frac{1}{2}\theta^2 - \xi\theta + \xi^2$$

in $H^4(C^{(4)}, \mathbb{Z})$. Using [7, (15.1)] we have $\pi_4^*\xi = f_1 + \ldots + f_4$ and for the main diagonal $\delta = \delta(2^1, 1^2)$ in the symmetric product $C^{(4)}$ we have obviously $\pi_4^*\delta = \sum_{i<j} \Delta_{ij}$. Combining this with $\theta = 6\xi - \frac{1}{2}\delta$, which follows from [7, (15.4)] (note that $\xi = \eta$ and $\theta = \sigma_1 + \ldots + \sigma_g$ translates our and Macdonald's notation), we obtain $\pi_4^*\theta = 6\sum f_i - \sum_{i<j} \Delta_{ij}$. In order to determine the class of V, we then compute

$$\begin{aligned} (\sum f_i)^2 &= 2\sum_{i<j} f_i f_j \\ (\sum f_i) \cdot (\sum_{j<k} \Delta_{jk}) &= 2\sum_{i<j} f_i f_j + \sum_{j<k, i \neq j, i \neq k} f_i \Delta_{jk} \\ (\sum_{i<j} \Delta_{ij})^2 &= -4\sum_{i<j} f_i f_j + 6\sum_{i<j<k} \Delta_{ijk} + 2(\Delta_{12}\Delta_{34} + \Delta_{13}\Delta_{24} + \Delta_{14}\Delta_{23}) \end{aligned}$$

and we use again the (pull-back to C^4 of the) relation [7, (15,4)] (with $s = 3$ this time)

$$\sum_{j<k, i\neq j, i\neq k} f_i\Delta_{jk} = 2\sum_{i<j} f_if_j + \sum_{i<j<k} \Delta_{ijk} \ .$$

This leads us to

$$[V] = \left(4\sum_{i<j} f_if_j - 2\sum_{i<j<k}\Delta_{ijk} + \Delta_{12}\Delta_{34} + \Delta_{13}\Delta_{24} + \Delta_{14}\Delta_{23}\right)\cdot\Delta_{34} \ .$$

Taking the image of V under pr_{12} corresponds to projecting onto the first factor in the Knneth decomposition

$$H^6(C^4) \cong H^2(C^2)\otimes H^4(C^2) \oplus H^3(C^2)\otimes H^3(C^2) \oplus H^4(C^2)\otimes H^2(C^2) \ .$$

Therefore, the class of G is symmetric and it is contained in the subgroup of $H^2(C^2,\mathbb{Z})$ generated by the sum of the fibers, $F_1 + F_2$, and the diagonal Δ. So we can write

$$[G] = \alpha[F_1 + F_2] - \beta\Delta$$

with integer coefficients α and β. Since C is not hyperelliptic, the projection $pr_{12}|_V$ is birational onto its image, hence

$$\begin{aligned} \alpha - \beta &= G\cdot F_i \\ &= (pr_{12})_*V\cdot F_i \\ &= W\cdot\Delta_{34}\cdot f_i \\ &= \pi_4^*C_4^2\cdot\Delta_{34}\cdot f_i \\ &= C_4^2\cdot\pi_{4*}(\Delta_{34}\cdot f_i) \ . \end{aligned}$$

The image of $\Delta_{34}\cdot f_i$ under π_4 is just $j_3(\delta(2,1))$ and the map is obviously of degree one, thus

$$\alpha - \beta = C_4^2\cdot j_3(\delta(2,1)) = \left(\frac{1}{2}\theta^2 - \xi\theta + \xi^2\right)\cdot(10\xi^2 - 2\theta\xi) = 10 \ . \tag{4}$$

Similarly, we have

$$2\alpha + 4\beta = G\cdot\Delta = C_4^2\cdot\pi_{4*}(\Delta_{34}\cdot\Delta_{12}) \ .$$

Now, under π_4 the intersection $\Delta_{34}\cap\Delta_{12}$ maps $2:1$ to the diagonal $\delta(2^2)$. The class of this diagonal can be determined using [7, (15.2)], so that we obtain with a calculation

$$\begin{aligned} 2\alpha + 4\beta &= C_4^2\cdot 2\cdot\delta(2^2) \\ &= \left(\tfrac{1}{2}\theta^2 - \xi\theta + \xi^2\right)\cdot 2\cdot(28\xi^2 - 12\xi\theta + 2\theta^2) = 56 \ . \end{aligned} \tag{5}$$

In the last step one uses the Poincaré formula to determine the intersection numbers involving the classes θ and ξ. From (4) and (5) we find that

$$G \equiv 16(F_1 + F_2) - 6\Delta \ .$$

Since C is not hyperelliptic, the projections $pr_{12}|_V$ and $pr_{34}|V$ are of degree 1 and 2 respectively. Moreover, we have $V = (pr_{34}|_V)^*\Delta$, and hence G is either irreducible or a union of two irreducible curves G_1, G_2, both isomorphic to C and symmetric to each other with respect to ι. In the second case $p_1|_{G_1}$ is a degree 10 endomorphism of C or $p_1(G_1)$ is a point. The first is impossible and the second contradicts the numerical class of G. Hence G is irreducible and Proposition 1.2 applies. □

Remark 2.1 The second part of the above proof works also in the hyperelliptic case. However, in that case pr_{34} is no more finite. It has 1-dimensional fibers over the Weierstraß points of Δ; they are the graphs of the hyperbolic involution. If $p \in C$ is not a Weierstraß point, then $pr_{34}^{-1}(p,p) = (\iota(p), \iota(p), p, p)$, so that applying $(pr_{12})_*$ we get 2Δ. Thus G is non-reduced and reducible: $G \equiv 8(2(F_1 + F_2) - \Delta) + 2\Delta$.

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