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Fostering subject-driven professional competence of pre-service mathematics teachers – a course conception and first results

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Abstract: We present ProfiWerk, a professionalization course geared towards pre-service Gymnasium teachers in mathematics, which is part of the preparation for an extended school-internship phase. Since the transition from university education to school practice can come with adverse discontinuity effects – rendering, at worst, university education ineffective – special focus is put on establishing stable connections between both mathematics content knowledge and mathematics education knowledge to the professional demands on mathematics teachers.

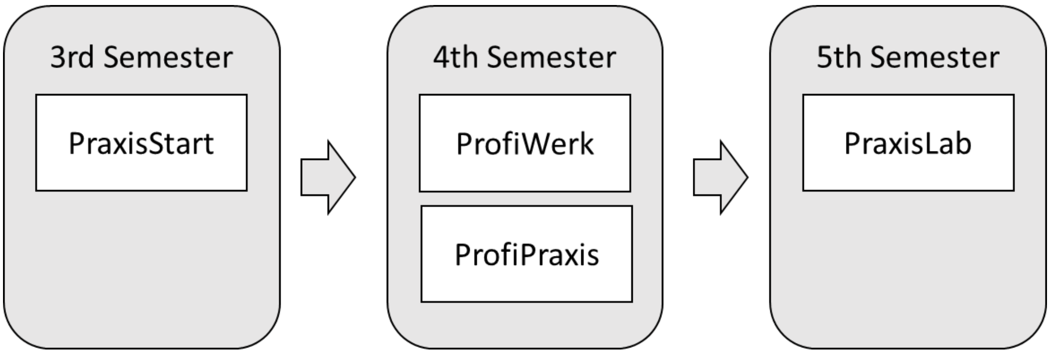
Introduction

Prolonged school internships of several months are increasingly becoming an integral part of pre-service mathematics teacher education in Germany. One of their aims is to improve the quality of university pre-service teacher education through providing opportunities for students to gain realistic work experience, while putting mathematics content knowledge, mathematics education knowledge as well as general pedagogical knowledge into action. It would be unreasonable to expect that these kinds of effects occur automatically as a necessary consequence of in-school experience. Rather, it will be crucial to establish stable links between knowledge acquired within university courses and the professional demands on teachers (cf. Kuntze et al. 2009). There is reason for concern that the well-known problem of *double discontinuity* (Klein 1908) might occur, if these links are not sufficiently established. Just as many novice teachers may not consider knowledge that they acquired at university particularly relevant for their teaching, students in an internship might tend to plan and implement their teaching attempts in an ad-hoc fashion, rather than based on theories studied at university. There is a tendency for novice teachers to regard the latter as unworkable in the "real world" of the classroom

(Cavanagh and Prescott 2007). They are then susceptible to the “familiarity pitfall” (Feiman-Nemser and Buchmann 1985, p. 56), i.e., they might identify teaching with classroom practices that they experienced as pupils themselves. This is what Felix Klein considered a consequence of the “second discontinuity”.

At Philipps-Universität Marburg, a curriculum that includes a 2-month internship during the semester („Praxissemester“) is part of the project *ProPraxis* (see Laging et al. 2015)-. In order to address the concerns described above, the project takes a comprehensive approach that encompasses several modules building on one another:

1. *PraxisStart* – a two-week classroom experience focusing on observation, along with a pedagogy seminar;
2. *ProfiWerk* (subject-specific professionalization workshop) – a unit that aims to link content knowledge and pedagogical content knowledge to the professional demands on prospective teachers (the mathematics incarnation of this unit will be presented in detail in this article);
3. *ProfiPraxis* – a pedagogy course preparing the school internship;
4. *PraxisLab* – the school internship, accompanied by a mathematics education course and a pedagogy course.



Sequence of modules leading into the school internship in the ProPraxis project

This sequence of modules is driven by what in project terminology is called the *twofold understanding of practice*:

- The *first understanding of practice* means to experience oneself authentically as a practitioner of mathematics, thus experiencing, deepening and appreciating subject specific-content, world views and approaches. Reflecting on these aspects and on their role for teaching this subject is conceptualized as a key to subject-driven professional competence.
- In a second step, within the school-internship module *PraxisLab*, this first view of practice is linked to a *second understanding of practice* in the sense of actual teaching practice.

The feature that sets *ProPraxis* most apart from other conceptions for prolonged school internships is arguably the fact that in addition to a mathematics education course that accompanies the internship, there is the professionalization workshop *ProfiWerk Fach* (*ProfiWerk* for short), taking place right before the internship phase, which puts special emphasis on subject-driven professional competence. In this text, we will focus on the *ProfiWerk* project component in mathematics, presenting the concept of the course and describing first results from the pilot phase.

At the time of preparing this paper, the modules described above were at an experimental stage in 11 out of the 20 subject areas that students can choose in the teacher education program. In the meantime, these modules have been integrated as regular components of the Marburg teacher education program. Slight modifications were made during this process to the project as a whole, but these do not affect the conception of the ProfiWerk project component that we consider in this paper.

Professionalization Workshop ProfiWerk – The Course Concept

ProfiWerk aims at students experiencing, deepening, appreciating and critically reflecting on mathematics-specific content, methods, theory-building processes, world views and approaches. In the course, students should identify and reflect upon:

- key questions that initiate mathematical work,
- core activities of mathematical work,
- fundamental ideas in mathematics, and
- specific beliefs and attitudes towards those activities and ideas.

Concretely, we conceived the course as consisting of two sequences, the first of which focused on the fundamental idea of *number*, while the second focused on the core activity of *reasoning and proof*. Each of the two sequences starts by having the students work on the relevant subject matter knowledge at university and school level, with the aim of broadening their prospective on fundamental ideas and core activities of mathematical work. This activity on the object level is complemented on the meta-level by input from the philosophy and history of mathematics. In the next step, the subject matter is considered from the perspective of mathematics education, with school textbook excerpts, pupils' work, and the students' own working processes (from videography) on the object level, as well as theory from mathematics education on the meta level. In a final step, students work on elementarizing the considered contents, methods and theory-building processes, in a format suitable for teaching school mathematics. On the meta-level, they analyze authentic products by pupils, in particular from a trial implementation of student-designed tasks that was carried out by an experienced teacher (the third author).

The following *core matrix* represents the result of these considerations and serves as the structural foundation of each sequence:

	Subject-matter oriented broadening of horizon	**Theoretical input and application from mathematics education perspective**	**Concrete modelling work (in small groups)**
Object Level	Deepening selected mathematical content, methods, theory-building processes at university and school level	Pupils' work, school book excerpts, own working processes (videography)	Elementarizing and preparing these contents, methods, theory building processes in a suitable format for school
Meta Level	Input from the philosophy and history of mathematics	Concepts and theories from mathematics education	Analyzing pupil's products from a trial implementation (conducted by an experienced teacher)

We will describe below how the matrix is being implemented for the fundamental idea of *number* as well as for the core activity of *reasoning and proof.*

The Role of Reflection for Professionalization

General Idea

The role of reflection on the professional development and growth of teachers has been pointed out by many authors (see Schön 1983, and e.g. Scales 2012 for the concept of the *Reflective Teacher*). The main idea is that successful teachers – and successful practitioners in general – improve their expertise through an iterative process that consists of critically analyzing their work (what happened, how and why it happened) and by drawing conclusions from these experiences and analyses for future work.

In view of its importance, we made *reflection-on-action* tasks (Schön, 1983) a part of the regular course homework. These tasks demanded from the pre-service teachers to "consciously review, describe, analyse and evaluate" their experiences made during the course lessons or in homework activities, "with a view to gaining insight to improve future practice" (Finlay 2008, p. 3).

In order to provide feedback, we established a scheme of reflection levels as a guideline, which was also used for grading purposes. The formulation of these reflection levels follows the work of Schön on reflection-on-action (1983), and is compatible with various other models, e.g., Atkins' and Murphy's three stages of the reflective process (1993), as well as with the three fundamental processes "retrospection", "self-evaluation" and "reorientation" identified by Quinn (1988/2000) as common elements of different models of reflection (compare Finlay 2008 for an overview):

- *Reflection Level 1:* Students consider their experience from a distance, they evaluate their practice or the practice of others.
- *Reflection Level 2:* Students identify and articulate problems or the need for action or advancement (or the potential for such)
- *Reflection Level 3:* Students find and describe alternatives for action or possibilities for advancement for the identified problems and potentials.
- *Reflection Level 4:* Students articulate and reflect on experiences that they have already gained with action alternatives or with attempts to make use of the recognized potential for advancement.

Reflective Writing

The first unit of the course starts with an activity in reflective writing regarding the notion of "central mathematical ideas", whose purpose is twofold. On the one hand it provides the course instructors with an opportunity to observe the students' beliefs and attitudes on core mathematical activities and ideas. On the other hand, it helps in making the students themselves more aware of their beliefs and attitudes, thus rendering those more accessible for change or further development. For our course we extended the freewriting and clustering method from Frank et al. (2013, Chap. 3.2), so that the reflective writing activity in our course conception consisted of the following steps:

(1) **Clustering I:** In this first step, participants are asked to link spontaneous notions and associated concepts regarding "central mathematical ideas" via (written) clustering.[1]

(2) **Clustering II:** The second step aims at finding a personal focus: one's own position on what is essential about "central mathematical ideas". Participants are asked to have a second look at their initial cluster from step 1, and to supplement it arbitrarily until their attention is drawn into a certain direction. They shall grasp and formulate this focus as a core thought, and even build a new cluster around it as needed.

(3) **Freewriting I:** In a third step, running text is produced around the core thought worked out in step 2. The text product is only for private use, and free from formal requirements like correct grammar, spelling, language aesthetics etc.[2]

(4) **Freewriting II:** The fourth step, as the second with regard to the first, aims at focusing the product of the third step. Participants are asked to find and reformulate more concisely the central sentence of their first freewriting, and take this as a starting point for an iteration of the freewriting process. The resulting

[1] Clustering is a popular phase in writing process navigation. Most of the clustering methods used today are enhancements of Rico's methods initially developed in (Rico 1983).

[2] Freewriting is another standard element of writing process navigation. Freewriting methods are more or less further developments of the works of Elbow (e.g., 1973) and Macrorie (1985). We used a loosely-focused freewriting variant here.

text product is semi-private, as it is made available to fellow students in a partner reflection.

(5) **Bulls Eye:** The fifth step distills a core statement of the whole freewriting phase.

(6) **Partner reflection:** In a sixth step, participants comment on what they perceive to be the central statement of the second freewriting of their fellow students, and discuss this with the authoring student.

(7) **Public text:** The "Bulls Eye" statement is used as the central thesis of a public text written by the participants after the partner reflection phase, and is made available to external readers (in our case: the instructors).

Notice that for steps (1)-(4), it is essential to have sharp time frames of only a few minutes each, and for steps (3) and (4) in particular, it is necessary to avoid any pause or interruption of the writing process.

Some results from reflective writing. We show here samples from the public texts of three students. (We provide translations in addition to the original texts in German.) In the following text, the student focused on the role of proof in mathematics and in school, as well as on certain aspects of proofs such as their lengths. She expresses preferences concerning certain types of proofs:

> Proofs play an important role in mathematics […] Often, the length of a proof depends on the "knowledge" of the person carrying out the proof. The more she knows and understands about mathematics, the shorter she can keep the proof, because more "tools" are available. [...] Carrying out a proof is an ability that one has to learn and practice. [...] Personally, I like proofs where one calculates and transforms expressions more than proofs that use mathematical structures such as sets. I like it best when I can do a proof by induction. […] As proofs play such a big part at university, I find it desirable that some more proofs would be done in school.
>
> Beweise spielen in der Mathematik eine große Rolle. [...] Häufig hängt die Länge des Beweises mit dem ,,Wissen" des Beweisführenden zusammen. Je besser das Verständnis und die Kenntnisse über die Mathematik sind, desto eher ist man in der Lage sich kurz zu fassen, da einem mehr ,,Werkzeuge" zur Verfügung stehen. [...] Das Beweisen will erlernt und trainiert werden. [...] Ich persönlich ziehe Beweise bei denen gerechnet und umgeformt wird Beweisen vor, die sich zum Beispiel mathematischer Strukturen wie Mengen bedienen. Am besten gefällt es mir, wenn ich diesen Beweis induktiv führen kann, da die Induktion für mich die zugänglichste Beweisform ist. [...] Da Beweise in der Universität eine so große Rolle spielen, ist es in meinen Augen wünschenswert, wenn auch in der Schule etwas mehr bewiesen würde.

In the following text, mathematics appears as an ordering device with respect to "the ordinary", thus showing epistemological beliefs, attitudes, and emotions:

> If you look carefully, you can find mathematical structure in everyday life and one can really enjoy this moment of discovery. That's one of the rare moments where one doesn't feel inferior to the universe. Our part of the world likes rules

> and security. There are plenty of these in the immutable world of mathematics, and, and this makes mathematics the perfect and ever-present promoter of order in the world.
>
> Durch einen wachen Blick findet man im Alltäglichen mathematische Strukturen und kann den Moment der Offenlegung geradezu genießen. Für diesen besonderen Moment fühlt man sich dem Universum ausnahmsweise mal nicht unterlegen. Unser Teil der Welt liebt Richtlinien, Regeln und Sicherheiten. Diese findet man zu genüge in der unumstößlichen Mathematik, was sie für die Menschen zum perfekten und damit allgegenwärtigen Ordnungshelfer macht.

This student contrasts the algorithmic character of school mathematics with mathematical understanding, and links this to her past learning experiences:

> Even if teachers are good and competent, it is impossible to build deep mathematical understanding in school, let alone expand on it. Mathematics in school is for the most part concerned with algorithmic calculation and applications of mathematics. [...] We basically have to discover and learn it anew, in order to be able to later teach school mathematics – which is almost superficial – on the basis of this great expertise. It is of utmost importance that we give up the restricted and incomplete view of the subject that we experienced and possibly mastered at school, in order to make room for alternatives. These might not be closely held, but they are essential for pupils who have a different approach.
>
> Auch wenn die Lehrpersonen in der Schule gut und kompetent gewesen sind, kann in der Schule kein vertieftes mathematisches Verständnis auf- und noch weniger ausgebaut werden. Die Schule ist zum Großteil auf algorithmisches Rechnen und Anwendungsbezogenheit ausgelegt. [...] Wir müssen im Grunde die Mathematik neu entdecken und erlernen, um später in der Schule auf der Basis dieses großen Fachwissens die – fast oberflächliche – Schulmathematik unterrichten zu können. Dabei ist es unglaublich wichtig, dass wir den beschränkten und unvollständigen Blick auf die Thematik, mit dem wir sie in der Schule gesehen und möglicherweise gut beherrscht haben, verlieren, um Platz für die Alternativen zu schaffen, die einem persönlich vielleicht nicht sehr liegen, aber für Schülerinnen und Schüler, die eine andere Herangehensweise haben, essentiell wichtig sind.

The beliefs and attitudes towards mathematics that are expressed here give a rough impression of the variety of student's subjective mathematical landscapes. These vary in a spectrum spread between sophisticated conceptions and naïve, but strong views about what is the core (epistemological) character of mathematics. One challenge regarding our course conception is to make students aware of this variety, and of their own subjective viewpoints, as a basis for the intended reflections. In the following, we will refer back to selected aspects of the reported quotes.

Fundamental Idea: Number

The first part of the course focused on the fundamental idea of number. This allows us to consider the idea of its inner-mathematical “ordering” function, which complements the idea about mathematics as a tool for “ordering the world”, as expressed in one of the reflective writing texts above. It also provides an opportunity to focus and reflect on alternatives to “algorithmically dominated math lessons”, also brought up in a reflective writing text.

The core matrix for the first part of the course is as follows:

	Subject-matter oriented broadening of horizon	**Theoretical input and application from mathematics education perspective**	**Concrete modelling work**
Object Level	• Constructions of the real numbers vs. their axiomatic definition (as a text basis for the students, we used Ebbinghaus 1992) • Representation of decimal fractions as infinite series • Core facts about “$0,\overline{9} = 1$” and about non-repeating infinite decimal fractions	Pupils’ conceptions of non-terminating decimal numbers (as a text basis for the students, we used Bauer 2011)	Design of imaginary dialogue openings on infinite non-repeating decimal fractions and on the question “$0,\overline{9} = 1$?”
Meta Level	Input from history of mathematics (extensions of $\mathbb{N}, \mathbb{Z}, \mathbb{Q}$) (as a text basis for the students, we used Ebbinghaus 1992, Sfard 1991)	• Imaginary dialogues (as a text basis for the students, we used Wille 2013) • Structural vs. operational views of mathematical objects (as a text basis for the students, we used Sfard 1991)	Analyze results from trial implementation in classroom

“Subject-matter oriented broadening of horizon” and “theoretical input and application from mathematics education perspective” were pursued via text-based student's or instructor's presentation,[3] or via short and rather narrowly instructed single or group activities. Concrete modelling work was done in groups, with rather low guidance by instruction. In the following, we will elaborate on the two columns "theoretical input and application from mathematics education perspective" and "concrete modeling work" of our matrix with some illustrating examples.

Pupils' conceptions of infinite decimal numbers and the case of $0,\overline{9} = 1$. The perception that $0,\overline{9}$ is less than – instead of equal to – 1 is a frequently reported phenomenon regarding secondary school children, but also first year students at universities or colleges (cf. e.g. Tall, 1977; Tall and Schwarzenberger 1978; Monaghan, 2001; Eisenman, 2008). Since an important goal of ProfiWerk is to make preservice teachers sensitized for careful work with pupil's mathematical productions, such reports were a primary motivation for choosing this particular issue as a course theme from the range of the fundamental idea of number. Another reason for this choice was the idea that the rather intellectually demanding conception and powerful mathematical tool of the real numbers needs some reflection beyond the normally axiomatic introduction in a first semester analysis course to teach real number concepts coherently in school, and to develop sophisticated ideas about their mathematical nature. The latter particularly affects, e.g., ideas about an „immutable world of mathematics" that renders "perfect and ubiquitous help in ordering the world" that were expressed in some of the reflective writing texts.

Imaginary dialogues. As a special student activity, we used the idea of imaginary dialogues (Wille 2016; see also Müller-Hill and Wille 2018). These are a form of writing and communicating which as a reflection device bridge the gap between guided and free reflection. Learners are presented with a dialogue opening that concerns a mathematical question, and they are prompted to continue the dialogue. Imaginary dialogues can be used to initiate learning processes, and to analyze cognitive and reasoning processes. In our course, we used the idea in the following way:

1. Students prepared (in small groups) dialogue openings that should encourage reasoning on the following questions:
    a. *Do non-terminating, non-repeating decimal fractions exist?*
    b. *Does 0,9999... = 1?*
2. As a first step towards school practice (in project terminology it is a step from the first practice to the second practice), these dialogue openings were then given to pupils from a contact class (grade 6), who continued the dialogues.
3. The completed dialogues were then handed back to those student groups who had prepared the openings. They analyzed the completed dialogues and produced a collection of the pupils' own writing (*autographs* in the spirit of Ruf

[3] The references given in the matrix refer to the main texts that were given to the participants.

and Gallin 1998/99). These were divided into the categories "Heart of the matter", "Is it true?", "Their own ways", and "Further questions".

As an example, we show here two imaginary dialogue openings that students in our course prepared. The first dialogue concerns question (a):

S1	Remember? We learned how to transform common fractions into decimal fractions. For example, if I transform $\frac{1}{3}$ or $\frac{1}{7}$ into a decimal fraction, I get $\frac{1}{3} = 0,\overline{3}$ with period length 1, and $\frac{1}{7} = 0,\overline{142857}$ with period length 6.
S2	Period length? You mean for the case of $\frac{1}{3}$, one digit is always repeated, and for $\frac{1}{7}$, it is six digits?
S1	Exactly. I'm wondering how many digits a period can have.
S2	Do you think there are periods that are so long that they are not periods anymore?
S1	Hm...? You mean decimal fractions with infinitely many digits, but non-repeating?
S2	Yes. What would such numbers look like? Do you think we can make some?
S1	Let's give it a try!

The second dialogue concerns question (b): Does 0,9999... = 1 hold?

S1	Now I understand what periodic decimal fractions are! They are infinitely long and have repeating digits – but you're not allowed to round. Therefore, 0,999... and 1 are not equal.
S2	This is hard for me to imagine...
S1	Don't confuse me! I believe it's an approximation, but there is always something that fits in between, so 0,999... and 1 are not equal.
S2	Let's draw a number line...

A first analysis of some of the results of the implementation of these initial dialogues in the contact class can be found in (Müller-Hill and Wille 2018). Note that the student-written dialogue openings were intentionally not edited by the instructors, even though they do not meet the requirements for ideal dialogue openings as suggested, e.g., in Wille (2009). Rather, we had the pupils work on the unedited dialogues openings and used the opportunity to have the students reflect upon the results: What does the produced dialogue tell about the knowledge and the conceptions of the pupils? And how effective was the dialogue opening in spawning a dialogue that reveals these aspects?

Core Activity: Reasoning and Proof

For the second part of the course, we specified our core matrix for the core mathematical activity of reasoning and proof. The purpose of this part is to enrich the participants' view of the role of proof. Also, this gives a further opportunity to focus and reflect on alternatives to "algorithmically dominated" mathematics lessons.

<table>
<tr><th></th><th>Subject-matter oriented broadening of horizon</th><th>Theoretical input and application from mathematics education perspective</th><th>Concrete modelling work</th></tr>
<tr><td>Object Level</td><td>Three proving problems</td><td>• Pupils' written reasoning
• Operative proofs by pupils
• Video recordings from participants own proving processes</td><td>Design of imaginary dialogue openings on proving in mathematics</td></tr>
<tr><td>Meta Level</td><td>History of mathematics (on linear independence)

Logic and philosophy:
• Toulmin-model, abduction (as a text basis for the students, we used Toulmin 2003),
• contexts of discovery vs. justification</td><td>• Functions of proofs (as a text basis for the students, we used Hanna 2000)
• Operative proofs (as a text basis for the students, we used Wittmann 2014)
• Concept image and concept definition (as a text basis for the students, we used Tall and Vinner 1981 and Tall 1988)</td><td>Analyze results from a test implementation in classroom</td></tr>
</table>

For the case of reasoning and proof, text-based student's or instructor's presentations within the scope of "subject-matter oriented broadening of horizon" and "theoretical input and application from mathematics education perspective" were complemented by

1. largely autonomous proving activities by a subgroup of the participants. These activities were documented using videography. The video material served as a basis for transcription and theory-based in-depth analysis by all participants.
2. guided group work on concrete reasoning and operative proving productions by pupils of the contact class.

The concrete modelling work consisted again in designing and assessing imaginary dialogues, much as we have already described in some detail for the case of "number". We describe here the mathematical problems that were given to some of the participants in the videography session.

Using videography. The following proof problems were given to some of the students. These students worked in groups of three people. Each group worked on exactly one of the problems. Their proving activities were video-taped and made accessible to the other participants for transcription and further analysis and discussion.

(1) Let g and h be two non-parallel straight lines, let P be their intersection point and Q another point on g. Is there a circle tangent to g at Q and tangent to h?

(2) For $n > 2$, is there always a prime between n and $n!$?

(3) Can the "baking tray" below be used to derive a closed formula for the sum of squares $\sum_{k=1}^{n} k^2$?

There are several reasons for our use of videography. First, participants should become more aware of their role as practitioner of mathematics in general. Second, participants should identify particular guiding elements for their mathematical work that were put into play during the working phases and group discussions on the three proof problems. Third, by transcribing part of the video material, participants were able to have a close look at the micro-level of group work in proving activities. And fourth, the analysis based on the distinction between context of discovery and context of justification was meant to emphasize the problem-solving face of mathematical proof, and to point to various other functions of proof than mere verification. In terms of Schön's conception of the „reflective practitioner" (Schön 1983), the videography fostered *reflection on action* and *reflection in action*, but also what we might call *higher-order reflection* (i.e., reflecting on reflection in action).

Expanding on mathematical core activities

We focused here on the mathematical core activity of reasoning and proof. It should be mentioned that ProfiWerk has a second, complementing part (organized as a block seminar prior to the internship), where

- the theme “key questions and fundamental mathematical ideas” is complemented by the question “Which characteristical elements guide mathematical work?”, and
- the core activity “reasoning and proof” is embedded into “mathematical problem-solving in general”.

We will, however, not go into more detail here, as the block seminar is conceptually quite different from the unit described here and is therefore best considered separately.

Assessment and stumbling blocks

The final exam for the ProfiWerk course consisted of two major tasks, corresponding to the fundamental idea of *number* and to the core activity of *reasoning and proof*, respectively. In each of these two areas, every column of the core matrix was represented as a subtask. For example, students were asked to reconstruct an abductive argument from a pupil’s text, and they had to analyze pupils’ reasoning using the Toulmin scheme. As for the idea of number, they were required to reconstruct pupils’ conceptions on non-periodic decimals from imaginary dialogues.

Both during the course and on the exam, we identified several stumbling blocks for students. In the part of the course focusing on the idea of *number*, missing prerequisites turned out to be a major obstacle: when studying constructions of the real numbers, participants had difficulties with mathematical concepts such as *equivalence relation* and *equivalence class* that were relevant in most of the constructions. Similarly, when working on the question of whether 0,999... = 1 and on non-repeating infinite decimal fractions, missing knowledge from the theory of infinite series was an obstacle for further work. In the second part of the course, when working on *reasoning and proof*, we made the following observations:

- Students showed rather poor performance in constructing proofs on a problem-solving activity in all three videography groups – partly due to missing heuristics and *control* strategies (in the sense of Schoenfeld 1985, p. 15).
- Students had structural difficulties using the Toulmin scheme (finding the warrant or categorizing its logical status) and in distinguishing deduction and abduction (e.g. in pupil’s written answers).
- Students found it difficult to differentiate between contexts of discovery and contexts of justification (when presented with videography recordings or transcripts), possibly because the students themselves lacked sufficient understanding of quality criteria for mathematical justification.

Such difficulties have an impact on the learning goals of the module: The intended fostering of subject-driven professional competence by means of reflection (meta-level) suffers from missing action registers on the object level of doing mathematics.

First Evaluation Results and Discussion

In the following, we briefly summarize and discuss some results from a quantitative and qualitative evaluation of the course. The questionnaire-based evaluation was conducted during and after the last course session. It focused on learning objectives (as in Frank and Kaduk 2016). In the first part of the questionnaire, participants were asked to give their personal ratings on four domains:

(A) understanding the mathematical content and the processes of concept and theory formation at an authentic university level;

(B) in-depth understanding of the subject matter of school textbooks and other learning and teaching materials;

(C) analyzing the mathematical content of pupil's productions, as a basis for assessment and for further development; and

(D) critically reflecting subjective attitudes towards and beliefs about mathematics.

Each domain was rated with respect to four aspects, using a four-point Likert scale:

(1) How important are learning gains in these areas for you?
(2) Based on my content knowledge, I believe that I am able to cope with the domain-specific challenges in these areas.
(3) By participating in this course, I feel now more able to cope with such challenges than before.
(4) I am now more aware of domain-specific challenges in these areas.

For nearly all participants (18 out of 23 course participants took part in the evaluation), domains B and C were rated as most important regarding desired learning gains. These two domains were also rated highest regarding actual learning achievements (awareness aspect (4) and self-efficacy aspect (3)) through participation in the ProfiWerk course. Domains A and D were rated as slightly less important regarding desired learning gains, and were rated lower regarding the actual learning achievements through participating in the course.

As a fifth aspect, we asked participants about their experiences with the various activities they were confronted with during the course. They had to evaluate the helpfulness of each activity (e.g., written reflections, students' presentations, working in groups, video analysis, working with pupils' productions) regarding domains (A)-(D), again on a four-point Likert-scale. In a first analysis, the results on aspect (5) can be summarized as follows:

- The activities of doing group work and listening to input by instructors were rated as most helpful regarding domain (A).
- The activities of doing reflections tasks and working with school textbooks, as well as group work and homework with mathematics or didactics content, were rated as most helpful regarding domain (B).
- The activities of designing and assessing writing task productions for pupils (including imaginary dialogue openings) and working with other productions by

pupils, as well as doing group tasks or individual writing tasks on mathematics or didactics content, were rated as most helpful regarding domain (C).

- The activities of doing written homework with mathematics or didactics content and listening to input by instructors, as well as doing reflection tasks and designing and assessing writing task productions for pupils, were rated as most helpful regarding domain (D).

In a second part of the questionnaire, participants were asked to give open-ended comments on those domains for which they had rated aspects (3) or (4) with "rather does not apply" or "definitely does not apply". Only very few students gave open-ended comments here, which may not be representative for the rest of the course participants.

For the case of domain A, one participant wrote that she would have appreciated more detailed investigations of mathematical subject matter questions in the course lessons in combination with more explicit and detailed hints for previous self-study of relevant mathematical content.

For the case of domains B and C, one of the participants considered working with imaginary dialogues as a helpful and meaningful activity in general, in the sense that the corresponding activities contribute essentially to her *understanding* of pupil's "conceptions, misconceptions and general lines of thought". However, the given *time frames* for the activities were perceived as being insufficient. She also mentioned that these activities have the potential to find out about the *efficacy* of her own, creative design of initial dialogues in relation to what pupils made out of it. She criticized that, in her view, the reflective activity focused too much on an overview analysis of the continued dialogues. As a consequence, we are planning more in-depth analyses of selected dialogues for the next round of the course.

For the case of domain D, another participant mentioned that besides working with imaginary dialogues, the reflective writing activity fostered *awareness and reflection of personal beliefs* and attitudes towards mathematics as a subject (in her case, particularly regarding the role of proof), and of subjective mathematical core conceptions. However, she demands *more instructed reflection* to strengthen this effect: "In sum, my own notions of core concepts did not become clear to me and should have been reflected upon more (German original: *Meine eigenen Vorstellungen zu Kernbegriffen sind mir insgesamt nicht allzu bewusst geworden und hätten für mich mehr reflektiert werden können*)".

Regarding preservice teacher education, it is controversial to which degree reflection tasks should be open or instructed. A detailed discussion for the case of fundamental mathematical ideas, with a tendency to encourage less instructed reflection activities, can be found, e.g., in (Müller-Hill 2015, Sect. 6.3 and Sect. 7). In our case, increasing instructed reflection should be carefully balanced with respect to the intended learning objective (D), "critically reflecting subjective attitudes towards and beliefs about mathematics." As an example of more instructed reflection in such a balanced sense, we are providing in the second round of the course for each reflection task a special version of the general reflection level scheme, where the levels are specified for the concrete task at hand.

The evaluation data that we briefly described above regarding the question of how helpful specific course activities were with respect to the learning objectives, is certainly highly interesting and warrants more consideration in the form of a more in-depth quanitative and qualitative analysis of these evaluation results.

The concept of the ProfiWerk course described in this article has proven successful and is being developed further on a continuous basis. As already mentioned above, it is complemented by a second part which focuses on characteristical elements guiding the course of action in mathematical problem solving (see Bauer, Müller-Hill, Weber 2018).

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