

$$\mathfrak{U}|\mathfrak{l}\ni\mathfrak{l}=\underbrace{\hat{\mathfrak{l}}+I}_{\phantom{\hat{\mathfrak{l}}+I}}\overline{\hat{\mathfrak{l}}-I}^{-1}$$

$$\downarrow a$$

$$\mathfrak{U}|\mathfrak{l}\ni\hat{\mathfrak{l}}=\underbrace{\mathfrak{l}+I}_{\phantom{\mathfrak{l}+I}}\overline{\mathfrak{l}-I}^{-1}$$

$$\mathfrak{U}|\mathfrak{l}:=\frac{\mathfrak{l}\overset{\mathfrak{l}}{\longleftarrow}\underset{\text{lin}}{\mathcal{D}_{\mathfrak{l}}}_{\text{hull}}\subseteq\mathfrak{l}}{\mathcal{G}_{\mathfrak{l}}\subseteq\mathfrak{l}\times\mathfrak{l}:\bigwedge_{\mathfrak{l}:\hat{\mathfrak{l}}\in\mathcal{D}_{\mathfrak{l}}}\mathfrak{l}\bowtie\underbrace{\mathfrak{l}}_{\phantom{\mathfrak{l}}}\hat{\mathfrak{l}}+\underbrace{\mathfrak{l}}_{\phantom{\mathfrak{l}}}\bowtie\mathfrak{l}\hat{\mathfrak{l}}=0\Leftrightarrow\mathfrak{l}\subseteq-\mathfrak{l}^*}$$

$$\bigwedge_{\mathfrak{l}\in\mathcal{D}_{\mathfrak{l}}}\overline{\mathfrak{l}\mathfrak{l}\pm\mathfrak{l}}^2=\overline{\mathfrak{l}\mathfrak{l}}^2+\overline{\mathfrak{l}}^2$$

$$\text{LHS} \, = \, \underbrace{\mathfrak{l}\mathfrak{l}\pm\mathfrak{l}}_{\phantom{\mathfrak{l}\mathfrak{l}\pm\mathfrak{l}}}\bowtie\underbrace{\mathfrak{l}\mathfrak{l}\pm\mathfrak{l}}_{\phantom{\mathfrak{l}\mathfrak{l}\pm\mathfrak{l}}} = \mathfrak{l}\mathfrak{l}\bowtie\mathfrak{l}\mathfrak{l} + \overbrace{\mathfrak{l}\bowtie\mathfrak{l}\mathfrak{l} + \mathfrak{l}\mathfrak{l}\bowtie\mathfrak{l}}^{=0} + \mathfrak{l}\bowtie\mathfrak{l}$$

$$\mathfrak{R}_{\mathfrak{l}\pm I}\overset{\mathfrak{l}\pm I}{\overset{\text{bij}}{\longleftarrow}}\mathcal{D}_{\mathfrak{l}}$$

$$\overline{(\mathfrak{l}\pm I)^{-1}}\leqslant 1$$

$$0=\underbrace{\mathfrak{l}\pm I}_{\phantom{\mathfrak{l}\pm I}}\mathfrak{l}=\mathfrak{l}\mathfrak{l}\pm\mathfrak{l}\overset{\cdot 1}{\Rightarrow}\overline{\mathfrak{l}}^2=0\Rightarrow\mathfrak{l}=0$$

$$\hat{\mathfrak{l}}\in\mathfrak{R}_{\mathfrak{l}\pm I}\Rightarrow\bigvee_{\mathfrak{l}\in\mathcal{D}_{\mathfrak{l}}}\hat{\mathfrak{l}}=\mathfrak{l}\mathfrak{l}\pm\mathfrak{l}\Rightarrow\overline{(\mathfrak{l}\pm I)^{-1}\hat{\mathfrak{l}}}=\overline{\mathfrak{l}}\overset{\cdot 1}{\leqslant}\overline{\mathfrak{l}\mathfrak{l}\pm\mathfrak{l}}=\overline{\mathfrak{l}}^{\hat{\mathfrak{l}}}$$

$$\mathfrak{R}_{\hat{\mathfrak{l}}}=\mathfrak{R}_{\mathfrak{l}+I}\overset{\mathfrak{l}+I}{\longleftarrow}\mathcal{D}_{\mathfrak{l}}\overset{\overline{\mathfrak{l}-I}^{-1}}{\longleftarrow}\mathfrak{R}_{\mathfrak{l}-I}$$

$$\curvearrowright$$

$$\hat{\mathfrak{l}}=\underbrace{\mathfrak{l}+I}_{\phantom{\mathfrak{l}+I}}\overline{\mathfrak{l}-I}^{-1}$$

$$\mathfrak{R}_{\mathfrak{l}-I} \ni \hat{1} = \mathfrak{l} \, \mathfrak{l} - \mathfrak{l}$$

$$\mathfrak{l} \in \mathcal{D}_{\mathfrak{l}} \Rightarrow \hat{\mathfrak{l}} \, \hat{1} = \mathfrak{l} \, \mathfrak{l} + \mathfrak{l}$$

$$\hat{\mathfrak{l}} \text{ monometric } \overline{\hat{\mathfrak{l}} \hat{1}} = \overline{\hat{1}}$$

$$\overline{\hat{\mathfrak{l}} \hat{1}}^2 = \overline{\mathfrak{l} \, \mathfrak{l} + \mathfrak{l}}^2 = \overline{\mathfrak{l} \, \mathfrak{l} - \mathfrak{l}}^2 = \overline{\hat{1}}^2$$

$$\mathcal{G}_{\hat{\mathfrak{l}}} \subset \mathbb{1} \times \mathbb{1}$$

$$\mathcal{G}_{\hat{\mathfrak{l}}} \ni \mathfrak{l}_n\colon \hat{\mathfrak{l}} \, \mathfrak{l}_n \rightsquigarrow \mathfrak{l}\colon \mathfrak{l} \in \mathbb{1} \times \mathbb{1} \Rightarrow \mathfrak{l} \in \mathfrak{l}_n = \mathfrak{l} \, \mathfrak{l}_n - \mathfrak{l}_n$$

$$\mathfrak{l}_n \in \mathcal{D}_{\mathfrak{l}}$$

$$\mathfrak{l} \in \hat{\mathfrak{l}} \, \mathfrak{l}_n = \mathfrak{l} \, \mathfrak{l}_n + \mathfrak{l}_n \Rightarrow \mathfrak{l} \, \mathfrak{l}_n \rightsquigarrow \frac{\mathfrak{l} + \mathfrak{l}}{2}$$

$$\mathfrak{l}_n \rightsquigarrow \frac{\mathfrak{l} - \mathfrak{l}}{2} \xRightarrow{\mathcal{G}_{\mathfrak{l}} \text{ abg}} \frac{\mathfrak{l} - \mathfrak{l}}{2} \in \mathcal{D}_{\mathfrak{l}}$$

$$\mathfrak{l} \, \frac{\mathfrak{l} - \mathfrak{l}}{2} = \frac{\mathfrak{l} + \mathfrak{l}}{2} \Rightarrow \mathfrak{l} = \underbrace{\mathfrak{l} - I} \frac{\mathfrak{l} - \mathfrak{l}}{2} \in \mathcal{D}_{\hat{\mathfrak{l}}}$$

$$\hat{\mathfrak{l}} \, \mathfrak{l} = \underbrace{\mathfrak{l} + I} \frac{\mathfrak{l} - \mathfrak{l}}{2} = \mathfrak{l} \Rightarrow \mathfrak{l}\colon \mathfrak{l} \in \mathcal{G}_{\hat{\mathfrak{l}}} \text{ abg}$$

$$\text{defect spaces } \mathcal{D}_{\hat{\mathfrak{l}}}^{\perp} = \frac{\mathfrak{l} \in \mathcal{D}_{\hat{\mathfrak{l}}}^*}{\hat{\mathfrak{l}} \mathfrak{l} = \mathfrak{l}} \mathfrak{R}_{\hat{\mathfrak{l}}}^{\perp} = \frac{\mathfrak{l} \in \mathcal{D}_{\hat{\mathfrak{l}}}^*}{\hat{\mathfrak{l}} \mathfrak{l} = -\mathfrak{l}}$$

$$\mathfrak{l} \in \mathcal{D}_{\hat{\mathfrak{l}}}^{\perp} = \mathfrak{R}_{\underbrace{\mathfrak{l} - I}}^{\perp} \Rightarrow \bigwedge_{\mathfrak{l} \in \mathcal{D}_{\mathfrak{l}}} 0 = \mathfrak{l} \mathfrak{x} \overline{\mathfrak{l} - I} \mathfrak{l} = \mathfrak{l} \mathfrak{x} \underbrace{\mathfrak{l} \, \mathfrak{l} - \mathfrak{l}} = \mathfrak{l} \mathfrak{x} \underbrace{\mathfrak{l}} - \mathfrak{l} \mathfrak{x} \mathfrak{l} \Leftrightarrow \bigwedge_{\mathfrak{l} \in \mathcal{D}_{\mathfrak{l}}} \mathfrak{l} \mathfrak{x} \underbrace{\mathfrak{l}} = \mathfrak{l} \mathfrak{x} \mathfrak{l} \Rightarrow \mathfrak{l} \in \mathcal{D}_{\hat{\mathfrak{l}}}^*$$

$$\hat{\mathfrak{l}} \, \mathfrak{l} = \mathfrak{l}$$

$$\mathfrak{l} \in \mathfrak{R}_{\hat{\mathfrak{l}}}^{\perp} = \mathfrak{R}_{\underbrace{\mathfrak{l} + I}}^{\perp} \Rightarrow \bigwedge_{\mathfrak{l} \in \mathcal{D}_{\mathfrak{l}}} 0 = \mathfrak{l} \mathfrak{x} \overline{\mathfrak{l} + I} \mathfrak{l} = \mathfrak{l} \mathfrak{x} \underbrace{\mathfrak{l} \, \mathfrak{l} - \mathfrak{l}} = \mathfrak{l} \mathfrak{x} \underbrace{\mathfrak{l}} + \mathfrak{l} \mathfrak{x} \mathfrak{l} \Leftrightarrow \bigwedge_{\mathfrak{l} \in \mathcal{D}_{\mathfrak{l}}} \mathfrak{l} \mathfrak{x} \underbrace{\mathfrak{l}} = -\mathfrak{l} \mathfrak{x} \mathfrak{l} \Rightarrow \mathfrak{l} \in \mathcal{D}_{\hat{\mathfrak{l}}}^*$$

$$\hat{\mathfrak{l}} \, \mathfrak{l} = -\mathfrak{l}$$

$$\mathcal{D}_{\hat{\mathfrak{l}}} \subset \mathbb{1} \supset \mathfrak{R}_{\hat{\mathfrak{l}}}$$

$$\mathcal{D}_{\hat{\mathfrak{l}}} \ni \mathfrak{l}_n \rightsquigarrow \mathfrak{l} \in \mathbb{1} \xRightarrow{\text{Cau}} 0 \rightsquigarrow \overline{\mathfrak{l}_m - \mathfrak{l}_n} = \overline{\hat{\mathfrak{l}} \, \underbrace{\mathfrak{l}_m - \mathfrak{l}_n}} = \overline{\hat{\mathfrak{l}} \mathfrak{l}_m - \hat{\mathfrak{l}} \mathfrak{l}_n} \text{ Cau}$$

$$\Rightarrow \mathcal{G}_{\hat{\mathfrak{l}}} \ni \mathfrak{l}_n\colon \hat{\mathfrak{l}} \, \mathfrak{l}_n \rightsquigarrow \mathfrak{l}\colon \mathfrak{l} \in \mathcal{G}_{\hat{\mathfrak{l}}} \text{ abg} \Rightarrow \mathfrak{l} \in \mathcal{D}_{\hat{\mathfrak{l}}}$$

$$\mathfrak{I}=\hat{\mathfrak{L}}\;1$$

$$\mathfrak{R}_{\hat{\mathfrak{L}}}\ni \mathfrak{I}_n\rightsquigarrow \mathfrak{I}\in \mathbb{1}\Rightarrow \mathfrak{I}_n=\hat{\mathfrak{L}}\;1_n$$

$$1_n\in \mathcal{D}_{\hat{\mathfrak{L}}}$$

$$\overset{\Rightarrow}{\text{Cau}}\;0\rightsquigarrow \overline{\mathfrak{I}_m-\mathfrak{I}_n}=\overline{\hat{\mathfrak{L}}\underbrace{1_m-1_n}}=\overline{1_m-1_n}\;\text{Cau}\;\Rightarrow \mathcal{G}_{\hat{\mathfrak{L}}}\ni 1_n\colon \mathfrak{I}_n\rightsquigarrow 1\colon \mathfrak{I}\in \mathcal{G}_{\hat{\mathfrak{L}}}\;\text{abg}\;\Rightarrow 1\in \mathcal{D}_{\hat{\mathfrak{L}}}$$

$$\mathfrak{I}=\hat{\mathfrak{L}}\;1\in \mathfrak{R}_{\hat{\mathfrak{L}}}$$

$$\mathbb{1}=\mathcal{D}_{\hat{\mathfrak{L}}}\times \mathcal{D}_{\hat{\mathfrak{L}}}^{\perp}=\mathfrak{R}_{\mathfrak{L}_{-I}}\times \Big\{\hat{\mathfrak{L}}1=1\Big\}=\mathfrak{R}_{\hat{\mathfrak{L}}}\times \mathfrak{R}_{\hat{\mathfrak{L}}}^{\perp}=\mathfrak{R}_{\mathfrak{L}_{+I}}\times \Big\{\hat{\mathfrak{L}}\mathfrak{I}=-\mathfrak{I}\Big\}$$

$$\mathcal{D}_{\mathfrak{L}^*}=\mathcal{D}_{\mathfrak{L}}\times \mathcal{D}_{\hat{\mathfrak{L}}}^{\perp}\times \mathfrak{R}_{\hat{\mathfrak{L}}}^{\perp}$$

$$\hat{\mathfrak{L}}=\begin{bmatrix}-\mathfrak{L} & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & -I\end{bmatrix}$$

$$\text{Ex } \mathbb{1}\in \mathcal{D}_{\mathfrak{L}^*}\Rightarrow \bigvee \mathfrak{I}\in \mathcal{D}_{\mathfrak{L}}\sqsubset \mathcal{D}_{\mathfrak{L}^*}\ni 1\;\hat{\mathfrak{L}}\;1=1$$

$$\underbrace{I+\hat{\mathfrak{L}}}_{\mathbb{1}}\mathbb{1}=\underbrace{I-\mathfrak{L}}_{\mathbb{1}}\mathfrak{I}+1=\underbrace{I+\hat{\mathfrak{L}}}_{\mathbb{1}}\mathfrak{I}+1\Rightarrow \underbrace{I+\hat{\mathfrak{L}}}_{\mathbb{1}}\frac{1}{2}=1=\underbrace{I+\hat{\mathfrak{L}}}_{\mathbb{1}}\underbrace{\mathbb{1}-\mathbb{1}}_{\mathbb{1}}\Rightarrow \underbrace{I+\hat{\mathfrak{L}}}_{\mathbb{1}}\underbrace{\mathbb{1}-\mathfrak{I}-\frac{1}{2}}_{\mathbb{1}}=0\Rightarrow \mathfrak{I}=-\;\hat{\mathfrak{L}}\;\mathfrak{I}\in \mathfrak{R}_{\hat{\mathfrak{L}}}^{\perp}$$

$$\Rightarrow \mathcal{D}_{\mathfrak{L}^*}\ni \mathbb{1}=\mathfrak{I}+\frac{1}{2}+\mathfrak{I}\in \mathcal{D}_{\mathfrak{L}}+\mathcal{D}_{\hat{\mathfrak{L}}}^{\perp}+\mathfrak{R}_{\hat{\mathfrak{L}}}^{\perp}$$

$$\hat{\mathfrak{L}}\;\mathbb{1}=-\;\mathfrak{L}\;\mathfrak{I}+\frac{1}{2}-\mathfrak{I}$$

$$\text{Eind } 0=\mathfrak{I}+\frac{1}{2}+\mathfrak{I}\Rightarrow 0=\underbrace{I+\hat{\mathfrak{L}}}_{\mathbb{1}}\underbrace{\mathfrak{I}+\frac{1}{2}+\mathfrak{I}}_{\mathbb{1}}=\mathfrak{I}+\frac{1}{2}+\mathfrak{I}-\;\mathfrak{L}\;\mathfrak{I}+\frac{1}{2}-\mathfrak{I}=\underbrace{\mathbb{1}-\mathfrak{L}\;\mathbb{1}}_{\mathbb{1}}+1\in \mathcal{D}_{\hat{\mathfrak{L}}}\times \mathcal{D}_{\hat{\mathfrak{L}}}^{\perp}$$

$$\Rightarrow \underbrace{I-\mathfrak{L}}_{\mathbb{1}}\mathfrak{I}=0=1\;\overset{\Rightarrow}{I-\mathfrak{L}}_{\text{inj}}\;\mathfrak{I}=0\Rightarrow \mathfrak{I}=0$$

$$\hat{\mathfrak{L}}=-\;\mathfrak{L}\Leftrightarrow \hat{\mathfrak{L}}\in \mathsf{U}|\mathbb{1}\;\text{unitary}$$

$$\mathbb{1}\;\overset{\supset}{\text{hull}}\;\mathfrak{R}_{\hat{\mathfrak{L}}_{-I}}=\mathcal{D}_{\mathfrak{L}}\overset{\hat{\mathfrak{L}}-I}{\underset{\text{bij}}{\leftarrow}}\;\mathfrak{R}_{\mathfrak{L}_{-I}}=\mathcal{D}_{\hat{\mathfrak{L}}}$$

$$\hat{\mathfrak{L}}\in \mathsf{U}|\mathbb{1}$$

$$\mathfrak{R}_{\mathfrak{l}} = \mathfrak{R}_{\hat{\mathfrak{l}}+I} \xleftarrow{\hat{\mathfrak{l}}+I} \mathcal{D}_{\hat{\mathfrak{l}}} \xleftarrow{\widehat{\mathfrak{l}-I}^{-1}} \mathfrak{R}_{\hat{\mathfrak{l}}-I} = \mathcal{D}_{\mathfrak{l}}$$


$$\mathfrak{l} = \underbrace{\hat{\mathfrak{l}}+I}_{\mathfrak{l}} \widehat{\mathfrak{l}-I}^{-1}$$

$$1 \in \mathcal{D}_{\hat{\mathfrak{l}}} \Rightarrow 1 = \mathfrak{l} \mathfrak{l} - \mathfrak{l}$$

$$\mathfrak{l} \in \mathcal{D}_{\mathfrak{l}} \Rightarrow \underbrace{\hat{\mathfrak{l}}-I}_{\mathfrak{l}} 1 = \hat{\mathfrak{l}} \ 1 - 1 = \underbrace{\mathfrak{l} \ \mathfrak{l} + \mathfrak{l}}_{\mathfrak{l}} - \underbrace{\mathfrak{l}_{\mathfrak{l}} - \mathfrak{l}}_{\mathfrak{l}} = 2\mathfrak{l} \Rightarrow \mathfrak{R}_{\hat{\mathfrak{l}}-I} = \mathcal{D}_{\mathfrak{l}}$$

$$\underbrace{\hat{\mathfrak{l}}-I}_{\mathfrak{l}} 1 = 0 \Rightarrow \mathfrak{l} = 0 \Rightarrow 1 = \mathfrak{l} \ \mathfrak{l} - \mathfrak{l} = 0 \text{ inj}$$

$$\underbrace{\hat{\mathfrak{l}}+I}_{\mathfrak{l}} \widehat{\mathfrak{l}-I}^{-1} \mathfrak{l} = \underbrace{\hat{\mathfrak{l}}+I}_{\mathfrak{l}} \frac{1}{2} = \frac{\hat{\mathfrak{l}} 1 + 1}{2} = \frac{\mathfrak{l} \ \mathfrak{l} + \mathfrak{l} + \mathfrak{l} \ \mathfrak{l} - \mathfrak{l}}{2} = \mathfrak{l} \ \mathfrak{l}$$