$$\begin{cases} m \cdot 0 = 0 \\ m \cdot n^+ = m \cdot n + m \end{cases} \overset{\text{mult}}{\Longrightarrow} \mathbb{N} \times \mathbb{N} \xrightarrow{\cdot} \mathbb{N}$$

$$m \cdot n \underset{\text{komm}}{=} n \cdot m$$

$$m^+ \cdot n = m \cdot n + n$$

$$0 = n: \quad m^+ \cdot 0 = 0 = 0 + 0 = m \cdot 0 + 0$$

$$0 \leqslant n \curvearrowright n^+: \quad m^+ \cdot n^+ = m^+ \cdot n + m^+ \underset{\text{Vor}}{=} \underbrace{m \cdot n + n} + m^+ \underset{\text{ass}}{=} m \cdot n + \underbrace{n + m^+} = m \cdot n + \underbrace{n^+ + m}$$

$$\underset{\text{komm}}{=} m \cdot n + \underbrace{m + n^+} \underset{\text{ass}}{=} \underbrace{m \cdot n + m} + n^+ = m \cdot n^+ + n^+$$

$$0 \cdot n = 0$$

$$0 = n: \quad 0 \cdot 0 = 0$$

$$0 \leqslant n \curvearrowright n^+: \quad 0 \cdot n^+ = 0 \cdot n + 0 \underset{\text{Vor}}{=} 0 + 0 = 0$$

$$0 = n: \quad m \cdot 0 = 0 = 0 \cdot m$$

$$0 \leqslant n \curvearrowright n^+: \quad m \cdot n^+ = m \cdot n + m \underset{\text{Vor}}{=} n \cdot m + m = n^+ \cdot m$$

$$\underbrace{a \cdot b} \cdot c \underset{\text{assoc}}{=} a \cdot \underbrace{b \cdot c}$$

$$\begin{cases} \prod_0^k a_k = 1 \\ \prod_{n+1}^k a_k = \underbrace{\sum_m^n a_k}\, a_n \end{cases} \underset{\text{product}}{\overset{\text{finite}}{\Longrightarrow}} \prod_m^n a_k$$

$$\begin{cases} x^0 = 1 \\ x^{n+1} = x^n \cdot x \end{cases} \overset{\text{power}}{\Longrightarrow} \quad x^n = \prod_m^n x$$

$$\begin{cases} 0! = 1 \\ \underbrace{n+1}! = n!n \end{cases} \overset{\text{unomial}}{\Longrightarrow} \quad n! = 1 \mathrel{\therefore} n$$

$$\begin{cases} \binom{n}{0} = 1 \\ \binom{n+1}{m} = \binom{n}{m} + \binom{n}{m-1} \end{cases} \overset{\text{binomial}}{\Longrightarrow} \quad \binom{n}{m}$$

$$a \cdot b = b \cdot a \underset{\text{Binomi}}{\Longrightarrow} \overbrace{a+b}^{n} = \sum_{0 \leqslant k \leqslant n} \binom{n}{k} a^{k} b^{n-k}$$

$$\sum_{1 \leqslant k \leqslant n} \begin{cases} k & = \dfrac{n\overbrace{n+1}}{2} \\ k^2 & = \dfrac{n\overbrace{n+1}\overbrace{2n+1}}{6} \\ k^3 & = \dfrac{n^2\overbrace{n+1}^{2}}{4} = \overbrace{\sum_{1 \leqslant k \leqslant n} k}^{2} \end{cases}$$

$$0 \leqslant n \curvearrowright n+1: \quad \text{LHS}_{n+1} = \sum_{1 \leqslant k \leqslant n+1} k^3 = \sum_{1 \leqslant k \leqslant n} k^3 + \overbrace{n+1}^{3}$$

$$\underset{\text{ind}}{=} \frac{n^2\overbrace{n+1}^{2}}{4} + \overbrace{n+1}^{3} = \overbrace{n+1}^{2} \underbrace{\frac{n^2}{4} + n + 1} = \overbrace{n+1}^{2} \frac{n^2 + 4n + 4}{4} = \frac{\overbrace{n+1}^{2}\overbrace{n+2}^{2}}{4} = \text{RHS}_{n+1}$$

$$n = \{0 : \cdots : n-1\}$$

$$\sum_k^n x^k = \frac{1 - x^k}{1 - x}$$

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

$$1 \leqslant k \leqslant n \Longrightarrow \binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1} = \binom{n}{k} + \binom{n}{k-1}$$