

$$\dots \xleftarrow{d} \mathbb{I}^{n+1} \xleftarrow{d} \mathbb{I}^n \xleftarrow{d} \mathbb{I}^1 \xleftarrow{d} \mathbb{I}^0 \xleftarrow[\text{res}]{} \underset{\text{inj}}{\mathbb{I}}$$

$$\square \text{ left cov } \begin{cases} \dots \xleftarrow{d} \mathbb{I}^{n+1} \square \xleftarrow{d} \mathbb{I}^n \square \xleftarrow{d} \mathbb{I}^1 \square \xleftarrow{d} \mathbb{I}^0 \square \xleftarrow[\text{res}]{} \underset{\text{inj}}{\mathbb{I}} \square \\ \mathbb{I} \blacksquare^n = \mathbb{I}^n \blacksquare = \widehat{\mathbb{I}^n \square} \sqcap \widehat{\mathbb{I}^n \square} \end{cases}$$

$$\nabla_{\mathbb{I}} \text{ left cov } \begin{cases} \dots \xleftarrow{d} \mathbb{I}^{n+1} \nabla_{\mathbb{I}} \xleftarrow{d} \mathbb{I}^n \nabla_{\mathbb{I}} \xleftarrow{d} \mathbb{I}^1 \nabla_{\mathbb{I}} \xleftarrow{d} \mathbb{I}^0 \nabla_{\mathbb{I}} \xleftarrow[\text{res}]{} \underset{\text{inj}}{\mathbb{I}} \nabla_{\mathbb{I}} \\ \mathbb{I} \nabla_{\mathbb{I}}^n = \mathbb{I}^n \nabla_{\mathbb{I}} = \widehat{\mathbb{I}^n \nabla_{\mathbb{I}}} \sqcap \widehat{\mathbb{I}^n \nabla_{\mathbb{I}}} \end{cases}$$

$$\square \text{ right contra } \begin{cases} \dots \xrightarrow{d} \mathbb{I}^{n+1} \square \xrightarrow{d} \mathbb{I}^n \square \xrightarrow{d} \mathbb{I}^1 \square \xrightarrow{d} \mathbb{I}^0 \square \xrightarrow[\text{res}]{} \underset{\text{proj}}{\mathbb{I}} \square \\ \mathbb{I} \blacksquare^n = \mathbb{I}^n \blacksquare = \widehat{\mathbb{I}^n \square} \sqcap \widehat{\mathbb{I}^n \square} \end{cases}$$

$$\mathbb{R} \triangleleft \widehat{\mathbb{K} \nabla_{\square}} \text{ right contra } \begin{cases} \dots \xrightarrow{d} \mathbb{R} \triangleleft \widehat{\mathbb{K} \nabla_{\mathbb{I}^{n+1}}} \xrightarrow{d} \mathbb{R} \triangleleft \widehat{\mathbb{K} \nabla_{\mathbb{I}^n}} \xrightarrow{d} \mathbb{R} \triangleleft \widehat{\mathbb{K} \nabla_{\mathbb{I}^1}} \xrightarrow{d} \mathbb{R} \triangleleft \widehat{\mathbb{K} \nabla_{\mathbb{I}^0}} \xrightarrow[\text{res}]{} \underset{\text{proj}}{\mathbb{R} \triangleleft \widehat{\mathbb{K} \nabla_{\mathbb{I}}}} \\ \mathbb{R} \triangleleft \widehat{\mathbb{K} \nabla_{\square}} = \mathbb{R} \triangleleft \widehat{\mathbb{K} \nabla_{\mathbb{I}^n}} = \mathbb{R} \triangleleft \widehat{\mathbb{K} \nabla_{\mathbb{I}^1}} \sqcap \mathbb{R} \triangleleft \widehat{\mathbb{K} \nabla_{\mathbb{I}^n}} \end{cases}$$