$$\mathbb{C}\,\blacktriangledown_{0}\,\mathbb{1} = \mho|\daleth \,\triangledown^{0}_{0}\,\mathbb{1} = \frac{\mho|\daleth \xleftarrow[\text{*-hom}]{\alpha} \mathbb{1}}{\daleth \underset{\alpha\ \text{fax}}{\overset{\text{abg}}{\sqsubset}} \daleth \curvearrowright \daleth = \begin{cases} 0 \\ \daleth \end{cases}}$$

$$\alpha \text{ irr } \Leftrightarrow {}^{\alpha}\mathbb{1}' = \frac{\daleth \in \mho|\daleth}{\bigwedge_{\daleth}^{\mathbb{1}} \daleth\ {}^{\alpha}\daleth = {}^{\iota}\daleth\daleth} = \mathbb{C}I$$

$$\Rightarrow: \quad \daleth \in {}^{\alpha}\mathbb{1}' \Rightarrow \daleth^{*}\ {}^{\alpha}\daleth = \overbrace{{}^{\iota}\daleth^{*}\daleth}^{*} = \overbrace{\daleth\ {}^{\alpha}\daleth^{*}}^{*} = {}^{\iota}\daleth\ \daleth^{*} \Rightarrow \daleth^{*} \in {}^{\alpha}\mathbb{1}' \underset{\text{OE}}{\Rightarrow} \daleth = \daleth^{*}$$

$$\underset{\text{SpecTh}}{\Rightarrow} \bigvee_{E_{\lambda}}^{W^{*}(\daleth) \subset {}^{\alpha}\mathbb{1}'} \daleth = \int_{d\lambda} E_{\lambda} \underset{\text{irred}}{\Rightarrow} E_{\lambda} = \begin{cases} 0 \\ I \end{cases} \Rightarrow \daleth \in \mathbb{C}I$$

$$\Leftarrow: \quad \daleth \underset{\alpha\ \text{fax}}{\overset{\text{abg}}{\sqsupset}} \daleth \Rightarrow \bigwedge_{\daleth}^{\daleth} \bigwedge_{\daleth'}^{\daleth^{\perp}} \daleth \ast \underbrace{{}^{\iota}\daleth\daleth'} = \underbrace{{}^{\alpha}\daleth^{*}\daleth} \ast \daleth' = 0 \Rightarrow {}^{\iota}\daleth\ \daleth' \in \daleth^{\perp}\ \alpha \text{ fax}$$

$$\Rightarrow {}^{\alpha}\mathbb{1}' \ni \daleth \xrightarrow[\text{proj}]{P} \daleth^{\perp} \Rightarrow P = \lambda I$$

$$\lambda^{2} = \lambda \Rightarrow \begin{cases} \lambda = 0 & \Rightarrow \daleth^{\perp} = 0 \\ \lambda = 1 & \Rightarrow \daleth^{\perp} = \daleth \end{cases}$$

$$\alpha{:}\daleth \text{ irred}$$

$$\daleth \in \daleth$$

$$\daleth \ast \daleth = 1 \Rightarrow \alpha{:}\daleth{:}\daleth \text{ cycl}$$

$$0 \neq {}^{\iota}\daleth\ \daleth \underset{\alpha\ \text{fax}}{\sqsubset} \daleth \Rightarrow {}^{\iota}\daleth\ \bar{\daleth} = \daleth$$

$$\text{cycl } \alpha{:}\urcorner \text{ irr} \iff \alpha_{\urcorner} \text{ pure}$$

$$\Longrightarrow: \quad 0 \leqslant \mathbb{L} \leqslant \alpha_{\urcorner} \underset{\text{LEM surj}}{\Longrightarrow} \bigvee \llcorner \in {}^{\alpha}\mathbb{I}' = \mathbb{C}I$$

$$0 \leqslant \llcorner \leqslant I$$

$$\mathbb{L} = \alpha_{\llcorner\,\urcorner} \Longrightarrow \llcorner = \lambda I$$

$$0 \leqslant \lambda \leqslant I \Longrightarrow \mathbb{L} = \alpha_{\llcorner\,\urcorner} = \lambda^{2} \alpha_{\urcorner} \Longrightarrow \alpha_{\urcorner} \text{ pure}$$

$$\Longleftarrow: \quad \underline{\urcorner} \overset{\text{abg}}{\underset{\alpha \text{ fax}}{\sqsupset}} \underline{\underline{\urcorner}} \Longrightarrow {}^{\alpha}\mathbb{I}' \ni \underline{\urcorner} \xrightarrow[\text{proj}]{P} \underline{\underline{\urcorner}} \Longrightarrow \alpha_{\urcorner} = \alpha_{P\urcorner} + \alpha_{(I-P)\urcorner} \geqslant \alpha_{P\urcorner} \geqslant 0$$

$$\underset{\text{pure}}{\Longrightarrow} \alpha_{P\urcorner} = \lambda^{2} \alpha_{\urcorner} = \alpha_{\lambda\urcorner} \underset{\text{LEM inj}}{\Longrightarrow} P = \lambda I \Longrightarrow \lambda = \begin{cases} 0 & \Longrightarrow \underline{\underline{\urcorner}} = 0 \\ 1 & \Longrightarrow \underline{\underline{\urcorner}} = \underline{\urcorner} \end{cases}$$