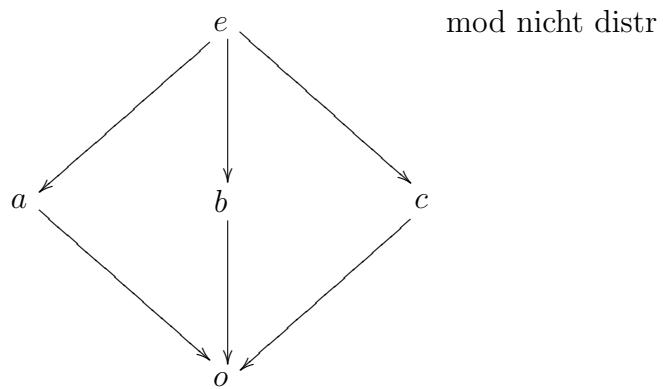


$$\mathbb{L} \text{ modular} \Leftrightarrow \bigwedge_{\mathbb{Y} \geqslant *} \mathbb{Y} \wedge \underbrace{* \vee *} = \underbrace{\mathbb{Y} \wedge *} \vee *$$

\mathbb{L} distr $\Rightarrow \mathbb{L}$ modular

$$\mathbb{Y} \geqslant * \Rightarrow \mathbb{Y} \wedge \underbrace{* \vee *} \underset{\mathbb{Y} \geqslant *}{=} \underbrace{\mathbb{Y} \vee *} \wedge \underbrace{* \vee *} \underset{\text{distr}}{=} \underbrace{\mathbb{Y} \wedge *} \vee *$$



\mathbb{L} modular $\Rightarrow \mathbb{L}$ modular

$$\mathbb{Y} \geqslant * \rightarrow \mathbb{Y} \leqslant * \rightarrow \mathbb{Y} \bar{\wedge} \underbrace{* \bar{\vee} *} = \mathbb{Y} \vee \underbrace{* \wedge *} = \underbrace{* \wedge *} \vee \mathbb{Y} \underset{\text{mod}}{=} * \wedge \underbrace{* \vee \mathbb{Y}} = * \bar{\vee} \underbrace{* \bar{\wedge} \mathbb{Y}} = \underbrace{\mathbb{Y} \bar{\wedge} *} \bar{\vee} *$$

$$\mathbb{M} \text{ modular} \Leftrightarrow \begin{cases} \mathbb{M} \geq \mathbb{M} \\ \mathbb{M} \wedge \mathbb{M} = \mathbb{M} \wedge \mathbb{M} \quad \mathbb{M} \wedge \mathbb{M} = \mathbb{M} \\ \mathbb{M} \vee \mathbb{M} = \mathbb{M} \vee \mathbb{M} \end{cases}$$

$$\Rightarrow : \mathbb{M} \underset{\text{Abs}}{=} \underbrace{\mathbb{M} \vee \mathbb{M}}_{\mathbb{M} \wedge \mathbb{M}} \wedge \mathbb{M} = \underbrace{\mathbb{M} \vee \mathbb{M}}_{\mathbb{M} \wedge \mathbb{M}} \wedge \mathbb{M} = \mathbb{M} \wedge \underbrace{\mathbb{M} \vee \mathbb{M}}_{\mathbb{M} \wedge \mathbb{M}} \underset{\text{mod}}{=} \underbrace{\mathbb{M} \wedge \mathbb{M}}_{\mathbb{M} \wedge \mathbb{M}} \vee \mathbb{M} = \underbrace{\mathbb{M} \wedge \mathbb{M}}_{\mathbb{M} \wedge \mathbb{M}} \vee \mathbb{M} \underset{\text{Abs}}{=} \mathbb{M}$$

$$\Leftarrow : \mathbb{M} \geq \mathbb{M} \Rightarrow \mathbb{M} \wedge \underbrace{\mathbb{M} \vee \mathbb{M}}_{\text{Lem}} \geq \underbrace{\mathbb{M} \wedge \mathbb{M}}_{\mathbb{M} \wedge \mathbb{M}} \vee \underbrace{\mathbb{M} \wedge \mathbb{M}}_{\mathbb{M} \wedge \mathbb{M}} = \mathbb{M} \vee \underbrace{\mathbb{M} \wedge \mathbb{M}}_{\mathbb{M} \wedge \mathbb{M}} \geq \mathbb{M} \wedge \mathbb{M} \Rightarrow \mathbb{M} \wedge \mathbb{M} = \mathbb{M} \wedge \underbrace{\mathbb{M} \wedge \mathbb{M}}_{\mathbb{M} \wedge \mathbb{M}} = \underbrace{\mathbb{M} \wedge \mathbb{M}}_{\mathbb{M} \wedge \mathbb{M}} \wedge \mathbb{M}$$

$$\leq \underbrace{\mathbb{M} \vee \underbrace{\mathbb{M} \wedge \mathbb{M}}_{\mathbb{M} \wedge \mathbb{M}}}^{\text{iso}} \wedge \mathbb{M} \leq \underbrace{\mathbb{M} \wedge \underbrace{\mathbb{M} \vee \mathbb{M}}_{\mathbb{M} \wedge \mathbb{M}}}^{\text{iso}} \wedge \mathbb{M} = \mathbb{M} \wedge \underbrace{\mathbb{M} \vee \mathbb{M} \wedge \mathbb{M}}_{\text{Abs}} \underset{\text{Abs}}{=} \mathbb{M} \wedge \mathbb{M}$$

$$\Rightarrow \mathbb{M} \wedge \mathbb{M} = \begin{cases} \overbrace{\mathbb{M} \vee \underbrace{\mathbb{M} \wedge \mathbb{M}}_{\mathbb{M} \wedge \mathbb{M}}}^{\mathbb{M} \wedge \mathbb{M}} \wedge \mathbb{M} \\ \mathbb{M} \wedge \underbrace{\mathbb{M} \vee \mathbb{M}}_{\mathbb{M} \wedge \mathbb{M}} \wedge \mathbb{M} \end{cases}$$

$$\text{dual } \mathbb{M} \vee \mathbb{M} = \mathbb{M} \bar{\wedge} \mathbb{M} = \begin{cases} \overbrace{\mathbb{M} \bar{\wedge} \underbrace{\mathbb{M} \bar{\wedge} \mathbb{M}}_{\mathbb{M} \bar{\wedge} \mathbb{M}}}^{\mathbb{M} \bar{\wedge} \mathbb{M}} \bar{\wedge} \mathbb{M} & = \overbrace{\mathbb{M} \wedge \underbrace{\mathbb{M} \vee \mathbb{M}}_{\mathbb{M} \wedge \mathbb{M}}}^{\mathbb{M} \wedge \mathbb{M}} \vee \mathbb{M} \\ \mathbb{M} \bar{\wedge} \underbrace{\mathbb{M} \bar{\wedge} \mathbb{M}}_{\mathbb{M} \bar{\wedge} \mathbb{M}} \bar{\wedge} \mathbb{M} & = \overbrace{\mathbb{M} \vee \underbrace{\mathbb{M} \wedge \mathbb{M}}_{\mathbb{M} \wedge \mathbb{M}}}^{\mathbb{M} \vee \mathbb{M}} \vee \mathbb{M} \end{cases}$$

$$\Rightarrow \begin{cases} \mathbb{M} \wedge \underbrace{\mathbb{M} \vee \mathbb{M}}_{\mathbb{M} \wedge \mathbb{M}} & \geq \mathbb{M} \vee \underbrace{\mathbb{M} \wedge \mathbb{M}}_{\mathbb{M} \wedge \mathbb{M}} \\ \underbrace{\mathbb{M} \wedge \underbrace{\mathbb{M} \vee \mathbb{M}}_{\mathbb{M} \wedge \mathbb{M}}}^{\mathbb{M} \wedge \mathbb{M}} \wedge \mathbb{M} & = \overbrace{\mathbb{M} \vee \underbrace{\mathbb{M} \wedge \mathbb{M}}_{\mathbb{M} \wedge \mathbb{M}}}^{\mathbb{M} \wedge \mathbb{M}} \wedge \mathbb{M} \underset{\text{Vor}}{\Rightarrow} \mathbb{M} \wedge \underbrace{\mathbb{M} \vee \mathbb{M}}_{\mathbb{M} \wedge \mathbb{M}} = \mathbb{M} \vee \underbrace{\mathbb{M} \wedge \mathbb{M}}_{\mathbb{M} \wedge \mathbb{M}} \\ \mathbb{M} \wedge \underbrace{\mathbb{M} \vee \mathbb{M}}_{\mathbb{M} \wedge \mathbb{M}} \vee \mathbb{M} & = \overbrace{\mathbb{M} \vee \underbrace{\mathbb{M} \wedge \mathbb{M}}_{\mathbb{M} \wedge \mathbb{M}}}^{\mathbb{M} \wedge \mathbb{M}} \vee \mathbb{M} \end{cases}$$