$$x^{u}\underline{g}^{\ddagger} = \underbrace{\overset{*}{a} + \overset{t}{b}\overset{*}{u}}x\underbrace{\bar{a} + \overset{*}{u}b}$$

$$\underbrace{\zeta\bar{a} + \bar{\zeta}}_{t}\underbrace{\zeta\bar{a} + \bar{\zeta}b} = \underbrace{\overset{*}{a} + \overset{t}{b}\overset{*}{u}}x\underbrace{\bar{a} + \overset{*}{u}b}$$

$$\text{LHS} = \underbrace{\overset{*}{a}\overset{t}{\zeta} + \overset{t}{b}\overset{*}{\zeta}}\underbrace{\zeta\bar{a} + \bar{\zeta}b} = \overset{*}{a}\underbrace{\overset{t}{\zeta}\zeta}_{=x}\bar{a} + \overset{*}{a}\underbrace{\overset{t}{\zeta}\bar{\zeta}}_{=x\overset{*}{u}}b + \overset{t}{b}\underbrace{\overset{*}{\zeta}\zeta}_{=\overset{*}{u}x}\bar{a} + \overset{t}{b}\underbrace{\overset{*}{\zeta}\bar{\zeta}}_{=\overset{*}{u}x\overset{*}{u}}b$$

$$= \overset{*}{a}x\bar{a} + \overset{*}{a}x\overset{*}{u}b + \overset{t}{b}\overset{*}{u}x\bar{a} + \overset{t}{b}\overset{*}{u}x\overset{*}{u}b = \text{RHS}$$

$$\{x\overbrace{\underbrace{T_u(g)}_{-1}\tilde{g}(u)}^{*}x\} = \{\underbrace{\overbrace{T_u(g)}^{\ast}\hat{g}(x)}\underbrace{\overbrace{\underbrace{T_u(g)}_{-1}\tilde{g}(u)}^{*}}\underbrace{\overbrace{T_u(g)}^{\ast}\hat{g}(x)}\} = \overbrace{T_u(g)}^{\ast}\{\hat{g}(x)\overbrace{\tilde{g}(u)}^{*}\hat{g}(x)\}$$

$$\gamma_z = \begin{array}{c|c} 0 & z \\ \hline \overset{*}{z} & 0 \end{array}$$

$$\gamma_z(w) = z - w\overset{*}{z}w$$

$$2\hat{\gamma}_z\left(\zeta\overset{t}{\zeta}\right) = \overset{*}{z}\bar{\zeta}\overset{t}{\zeta} + \zeta\overset{*}{\zeta}\overset{*}{z}$$

$$\gamma_z^{\mathbb{R}} = \begin{array}{c|c} 1 & 1 \\ \hline i & -i \end{array}\begin{array}{c|c} 0 & z \\ \hline \overset{*}{z} & 0 \end{array}\begin{array}{c|c} 1 & -i \\ \hline 1 & i \end{array} = \begin{array}{c|c} z + \overset{*}{z} & i(z - \overset{*}{z}) \\ \hline i(z - \overset{*}{z}) & -z - \overset{*}{z} \end{array} = \begin{array}{c|c} \alpha & \beta \\ \hline \gamma & \delta \end{array}$$

$$\alpha + i\gamma = \underbrace{z + \overset{*}{z}} - \underbrace{z - \overset{*}{z}} = \overset{*}{z}$$

$$\beta + i\delta = i\underbrace{z - \overset{*}{z}} - i\underbrace{z + \overset{*}{z}} = -i\overset{*}{z}$$

$$i\gamma - \delta = -\underbrace{z - \overset{*}{z}} + \underbrace{z + \overset{*}{z}} = \overset{*}{z}$$

$$\beta - i\alpha = i\underbrace{z - \overset{*}{z}} - i\underbrace{z + \overset{*}{z}} = -i\overset{*}{z}$$

$$\hat{\gamma}_z\left(\zeta\overset{t}{\zeta}\right) = \underbrace{\gamma_z\cdot\zeta}\overset{t}{\zeta} + \zeta\overbrace{\gamma_z\cdot\zeta}^{t} = \underbrace{\overset{*}{z}\xi - i\overset{*}{z}\eta}\overset{t}{\zeta} + \zeta\underbrace{\overset{t}{\xi}\overset{*}{z} - i\overset{t}{\eta}\overset{*}{z}} = \overset{*}{z}\underbrace{\xi - i\eta}\overset{t}{\zeta} + \zeta\underbrace{\overset{t}{\xi} - i\overset{t}{\eta}}\overset{*}{z}$$

$$\hat{\gamma}_z(u) = \hat{\gamma}_z\left(\vartheta\overset{t}{\vartheta}\right) = \overset{*}{z}\bar{\vartheta}\overset{t}{\vartheta} + \vartheta\overset{*}{\vartheta}\overset{*}{z} = \overset{*}{z}\overset{*}{u}u + u\overset{*}{u}\overset{*}{z} = \{\overset{*}{z}\overset{*}{u}u\}$$

$$\hat{\gamma}_z(x)\,\overset{*}{u}x + x\overset{*}{u}\,\hat{\gamma}_z(x) - 2x\,\overset{*}{\overbrace{uzu}}\,x = \overset{*}{z}\,\overset{*}{u}\,\underbrace{x\overset{*}{u}x} + \underbrace{x\overset{*}{u}x}\,\overset{*}{u}\,\overset{*}{z}$$

$$\hat{\gamma}_z(x) = \hat{\gamma}_z\left(\zeta\overset{t}{\zeta}\right) = \overset{*}{z}\,\overset{-}{\zeta}\,\overset{t}{\zeta} + \zeta\overset{*}{\zeta}\,\overset{*}{z} = \overset{*}{z}\,\overset{-}{\vartheta}\,\overset{2}{\lambda}\,\overset{t}{\vartheta} + \vartheta\overset{2}{\lambda}\,\overset{*}{\vartheta}\,\overset{*}{z}$$

$$x\overset{*}{u}x = \underbrace{\vartheta\overset{2}{\lambda}\overset{t}{\vartheta}}\,\underbrace{\overset{-}{\vartheta}\overset{*}{\vartheta}}\,\underbrace{\vartheta\overset{2}{\lambda}\overset{t}{\vartheta}} = \vartheta\overset{2}{\lambda}\,\underbrace{\overset{t}{\vartheta}\overset{-}{\vartheta}}\,\underbrace{\overset{*}{\vartheta}\vartheta}\,\overset{2}{\lambda}\,\overset{t}{\vartheta} = \vartheta\overset{4}{\lambda}\,\overset{t}{\vartheta}$$

$$\hat{\gamma}_z(x)\,\overset{*}{u}x + x\overset{*}{u}\,\hat{\gamma}_z(x) = \underbrace{\overset{*}{z}\overset{-}{\vartheta}\overset{2}{\lambda}\overset{t}{\vartheta} + \vartheta\overset{2}{\lambda}\overset{*}{\vartheta}\overset{*}{z}}\,\overset{-}{\vartheta}\,\overset{*}{\vartheta}\vartheta\overset{2}{\lambda}\,\overset{t}{\vartheta} + \vartheta\overset{2}{\lambda}\,\overset{t}{\vartheta}\,\overset{-}{\vartheta}\,\overset{*}{\vartheta}\,\underbrace{\overset{*}{z}\overset{-}{\vartheta}\overset{2}{\lambda}\overset{t}{\vartheta} + \vartheta\overset{2}{\lambda}\overset{*}{\vartheta}\overset{*}{z}}$$

$$= \underbrace{\overset{*}{z}\overset{-}{\vartheta}\overset{2}{\lambda}\overset{t}{\vartheta} + \vartheta\overset{2}{\lambda}\overset{*}{\vartheta}\overset{*}{z}}\,\overset{-}{\vartheta}\,\overset{2}{\lambda}\,\overset{t}{\vartheta} + \vartheta\overset{2}{\lambda}\,\overset{*}{\vartheta}\,\underbrace{\overset{*}{z}\overset{-}{\vartheta}\overset{2}{\lambda}\overset{t}{\vartheta} + \vartheta\overset{2}{\lambda}\overset{*}{\vartheta}\overset{*}{z}}$$

$$= \overset{*}{z}\,\overset{-}{\vartheta}\,\overset{2}{\lambda}\,\overset{t}{\vartheta}\,\overset{-}{\vartheta}\,\overset{2}{\lambda}\,\overset{t}{\vartheta} + 2\vartheta\overset{2}{\lambda}\,\overset{*}{\vartheta}\,\overset{*}{z}\,\overset{-}{\vartheta}\,\overset{2}{\lambda}\,\overset{t}{\vartheta} + \vartheta\overset{2}{\lambda}\,\overset{*}{\vartheta}\vartheta\overset{2}{\lambda}\,\overset{*}{\vartheta}\,\overset{*}{z} = \overset{*}{z}\,\overset{-}{\vartheta}\,\overset{4}{\lambda}\,\overset{t}{\vartheta} + 2\vartheta\overset{2}{\lambda}\,\overset{*}{\vartheta}\,\overset{*}{z}\,\overset{-}{\vartheta}\,\overset{2}{\lambda}\,\overset{t}{\vartheta} + \vartheta\overset{4}{\lambda}\,\overset{*}{\vartheta}\,\overset{*}{z}$$

$$\overset{*}{z}\,\overset{*}{u}\,\underbrace{x\overset{*}{u}x} + \underbrace{x\overset{*}{u}x}\,\overset{*}{u}\,\overset{*}{z} = \overset{*}{z}\,\overset{-}{\vartheta}\,\overset{*}{\vartheta}\,\underbrace{\vartheta\overset{4}{\lambda}\overset{t}{\vartheta}} + \underbrace{\vartheta\overset{4}{\lambda}\overset{t}{\vartheta}}\,\overset{-}{\vartheta}\,\overset{*}{\vartheta}\,\overset{*}{z} = \overset{*}{z}\,\overset{-}{\vartheta}\,\overset{4}{\lambda}\,\overset{t}{\vartheta} + \vartheta\overset{4}{\lambda}\,\overset{*}{\vartheta}\,\overset{*}{z}$$

$$\hat{\gamma}_z(x)\,\overset{*}{u}x + x\overset{*}{u}\,\hat{\gamma}_z(x) - \overset{*}{z}\,\overset{*}{u}\,\underbrace{x\overset{*}{u}x} - \underbrace{x\overset{*}{u}x}\,\overset{*}{u}\,\overset{*}{z} = 2\vartheta\overset{2}{\lambda}\,\overset{*}{\vartheta}\,\overset{*}{z}\,\overset{-}{\vartheta}\,\overset{2}{\lambda}\,\overset{t}{\vartheta}$$

$$x\,\overset{*}{\overbrace{uzu}}\,x = x\overset{*}{u}\,\overset{*}{z}\,\overset{*}{u}x = \underbrace{\vartheta\overset{2}{\lambda}\overset{t}{\vartheta}}\,\underbrace{\overset{-}{\vartheta}\overset{*}{\vartheta}\overset{*}{z}\overset{-}{\vartheta}\overset{*}{\vartheta}}\,\underbrace{\vartheta\overset{2}{\lambda}\overset{t}{\vartheta}} = \vartheta\overset{2}{\lambda}\,\underbrace{\overset{t}{\vartheta}\overset{-}{\vartheta}}\,\overset{*}{\vartheta}\,\overset{*}{z}\,\overset{-}{\vartheta}\,\underbrace{\overset{*}{\vartheta}\vartheta}\,\overset{2}{\lambda}\,\overset{t}{\vartheta} = \vartheta\overset{2}{\lambda}\,\overset{*}{\vartheta}\,\overset{*}{z}\,\overset{-}{\vartheta}\,\overset{2}{\lambda}\,\overset{t}{\vartheta}$$

$$x\,\overset{*}{\overbrace{\widetilde{\gamma}_z(u)}}\,x = x\,\underset{*}{\underbrace{z - u\overset{*}{z}u}}\,x$$

$$\widetilde{\gamma}_z(u) = u\overset{*}{u}z + z\overset{*}{u}u - u\overset{*}{z}u - u\,\underset{*}{\underbrace{u\overset{*}{z}u}}\,u = u\overset{*}{u}z + z\overset{*}{u}u - u\overset{*}{z}u - u\overset{*}{u}z\overset{*}{u}u$$

$$= \vartheta\overset{*}{\vartheta}z + z\overset{-}{\vartheta}\,\overset{t}{\vartheta} - \vartheta\overset{t}{\vartheta}\,\overset{*}{z}\vartheta\overset{t}{\vartheta} - \vartheta\overset{*}{\vartheta}z\overset{-}{\vartheta}\,\overset{t}{\vartheta}$$

$$x\,\overset{*}{\overbrace{\widetilde{\gamma}_z(u)}}\,x = \vartheta\overset{2}{\lambda}\,\overset{t}{\vartheta}\,\underset{*}{\underbrace{\vartheta\overset{*}{\vartheta}z + z\overset{-}{\vartheta}\overset{t}{\vartheta} - \vartheta\overset{t}{\vartheta}\overset{*}{z}\vartheta\overset{t}{\vartheta} - \vartheta\overset{*}{\vartheta}z\overset{-}{\vartheta}\overset{t}{\vartheta}}}\,\vartheta\overset{2}{\lambda}\,\overset{t}{\vartheta} = \vartheta\overset{2}{\lambda}\,\overset{t}{\vartheta}\,\underbrace{\overset{*}{z}\vartheta\overset{*}{\vartheta} + \overset{-}{\vartheta}\overset{t}{\vartheta}\overset{*}{z} - \overset{-}{\vartheta}\overset{*}{\vartheta}z\overset{-}{\vartheta}\overset{*}{\vartheta} - \overset{-}{\vartheta}\overset{t}{\vartheta}\overset{*}{z}\vartheta\overset{*}{\vartheta}}\,\vartheta\overset{2}{\lambda}\,\overset{t}{\vartheta}$$

$$= \vartheta\overset{2}{\lambda}\,\overset{t}{\vartheta}\,\overset{*}{z}\vartheta\overset{*}{\vartheta}\vartheta\overset{2}{\lambda}\,\overset{t}{\vartheta} + \vartheta\overset{2}{\lambda}\,\overset{t}{\vartheta}\,\overset{-}{\vartheta}\,\overset{t}{\vartheta}\,\overset{*}{z}\vartheta\overset{2}{\lambda}\,\overset{t}{\vartheta} - \vartheta\overset{2}{\lambda}\,\overset{t}{\vartheta}\,\overset{-}{\vartheta}\,\overset{*}{\vartheta}z\overset{-}{\vartheta}\,\overset{*}{\vartheta}\vartheta\overset{2}{\lambda}\,\overset{t}{\vartheta} - \vartheta\overset{2}{\lambda}\,\overset{t}{\vartheta}\,\overset{-}{\vartheta}\,\overset{t}{\vartheta}\,\overset{*}{z}\vartheta\overset{*}{\vartheta}\vartheta\overset{2}{\lambda}\,\overset{t}{\vartheta}$$

$$= \vartheta\overset{2}{\lambda}\,\overset{t}{\vartheta}\,\overset{*}{z}\vartheta\overset{2}{\lambda}\,\overset{t}{\vartheta} + \vartheta\overset{2}{\lambda}\,\overset{t}{\vartheta}\,\overset{*}{z}\vartheta\overset{2}{\lambda}\,\overset{t}{\vartheta} - \vartheta\overset{2}{\lambda}\,\overset{*}{\vartheta}z\overset{-}{\vartheta}\,\overset{2}{\lambda}\,\overset{t}{\vartheta} - \vartheta\overset{2}{\lambda}\,\overset{t}{\vartheta}\,\overset{*}{z}\vartheta\overset{2}{\lambda}\,\overset{t}{\vartheta} = \vartheta\overset{2}{\lambda}\,\overset{t}{\vartheta}\,\overset{*}{z}\vartheta\overset{2}{\lambda}\,\overset{t}{\vartheta} - \vartheta\overset{2}{\lambda}\,\overset{*}{\vartheta}z\overset{-}{\vartheta}\,\overset{2}{\lambda}\,\overset{t}{\vartheta}$$

$$x\,\underset{*}{\underbrace{u\overset{*}{z}u}}\,x = x\overset{*}{u}z\overset{*}{u}x = \vartheta\overset{2}{\lambda}\,\overset{t}{\vartheta}\,\overset{-}{\vartheta}\,\overset{*}{\vartheta}z\overset{-}{\vartheta}\,\overset{*}{\vartheta}\vartheta\overset{2}{\lambda}\,\overset{t}{\vartheta} = \vartheta\overset{2}{\lambda}\,\overset{*}{\vartheta}z\overset{-}{\vartheta}\,\overset{2}{\lambda}\,\overset{t}{\vartheta}$$

$$\hat{\gamma}_z(x)\,\overset{*}{u}x + x\overset{*}{u}\,\hat{\gamma}_z(x) + x\,\overset{*}{\overbrace{\widetilde{\gamma}_z(u)}}\,x = \overset{*}{z}\,\overset{*}{u}\,\underbrace{x\overset{*}{u}x} + \underbrace{x\overset{*}{u}x}\,\overset{*}{u}\,\overset{*}{z} + 2x\,\underset{*}{\underbrace{uzu}}\,x + x\overset{*}{z}x - x\,\underset{*}{\underbrace{u\overset{*}{z}u}}\,x$$