

$$\exists h \Leftrightarrow h^- * h \in \ker \nabla \Leftrightarrow h * h^- \in \ker \nabla$$

$$h * \ker \nabla = \frac{h * g}{g \in \ker \nabla} = \ker \nabla * h$$

$$\widehat{h * \ker \nabla} \oplus \widehat{h^- * \ker \nabla} = \widehat{h * h^-} * \ker \nabla$$

$$\mathbb{H} \cap \ker \nabla = \begin{cases} h * \ker \nabla \\ h \in \mathbb{H} \nabla \end{cases}$$

$${}^{h * \nabla_e} \mathbb{V} = {}^h \mathbb{V}$$

$$\begin{array}{ccccc}
\nabla_e & \xrightarrow{\text{ex}} & \mathbb{H} & \xrightarrow{\nabla} & \mathbb{K} \\
& \searrow 0 & \downarrow \text{on} & \nearrow \nabla \text{ inj} & \uparrow \text{in} \\
& & \mathbb{H} \cap \nabla_e & \xrightarrow{\tilde{\nabla}} & \mathbb{H} \nabla
\end{array}$$

$$\text{well-def : } \exists h \Rightarrow h^- * h \in \ker \nabla \Rightarrow h \nabla = \widehat{h \nabla} \circ e = \widehat{h \nabla} \circ \widehat{h^- * h \nabla}$$

$$= \widehat{h * h^- * h \nabla} = \widehat{h * h^-} * h \nabla = \widehat{e * h \nabla} = h \nabla$$

$$\mathbb{V} \text{ hom : } \widehat{h * \ker \nabla} \oplus \widehat{h^- * \ker \nabla} \mathbb{V} = \widehat{h * h^- * \ker \nabla} \mathbb{V} = \widehat{h * h^-} \nabla = \widehat{h \nabla} \circ \widehat{h^- \nabla} = \widehat{h * \ker \nabla} \mathbb{V} \circ \widehat{h^- * \ker \nabla} \mathbb{V}$$

$$\mathbb{V} \text{ inj : } \widehat{h * \ker \nabla} \mathbb{V} = \widehat{h^- * \ker \nabla} \mathbb{V} \Rightarrow h \nabla = h \nabla \Rightarrow \widehat{h^- * h \nabla} = \widehat{h^- \nabla} \circ \widehat{h \nabla} \\ = \widehat{h \nabla}^- \circ \widehat{h \nabla} = \widehat{h \nabla}^- \circ \widehat{h \nabla} = e \Rightarrow h^- * h \in \ker \nabla \Rightarrow \exists h \Rightarrow h * \ker \nabla = h * \ker \nabla$$

$$\tilde{\nabla} \text{ bij : } h \in \mathbb{H} \nabla \Rightarrow \bigvee_h h = h \nabla = \widehat{h * \ker \nabla} \mathbb{V} \Rightarrow h \in \mathbb{H} \cap \ker \nabla \mathbb{V}$$