$$\bar{\mathbb{R}}_{+} \underset{-}{\triangledown} \hbar = \frac{\bar{\mathbb{R}}_{+} \overset{\Vdash}{\longleftarrow} {}^{\hbar}\!\triangle 2}{P \subset \acute{P} \underset{\text{isoton}}{\Longrightarrow} \Vdash_{P} \leqslant \Vdash_{\acute{P}} : \quad \Vdash_{\bigcup\limits_{i}^{\mathbb{N}} P_i} \sigma \underset{\text{add}}{\overset{\text{sub}}{\leqslant}} \sum_{i}^{\mathbb{N}} \Vdash_{P_i}} \text{ out meas}$$

$$\mathcal{N} := \frac{\text{meas } N \subset \hbar}{\bigwedge\limits_{P \subset \hbar} \Vdash_{P} \geqslant \Vdash_{P \cap N} + \Vdash_{P \llcorner N}} = {}^{\hbar}\!\underset{\Vdash}{\triangle} 2 \Longrightarrow \mathcal{N} \text{ voll abz alg } \nu := \Vdash | \mathcal{N} \text{ voll meas}$$

$$\dot{N} \in \mathcal{N} \Longrightarrow \Vdash_{P \cap \underbrace{N \cup \acute{N}}} + \Vdash_{P \llcorner \underbrace{N \cup \acute{N}}} \leqslant \overbrace{\Vdash_{P \cap N} + \Vdash_{\underbrace{P \llcorner N} \cap \acute{N}}} + \Vdash_{\underbrace{P \llcorner N} \llcorner \acute{N}} \leqslant \Vdash_{P \cap N} + \Vdash_{P \llcorner N} \leqslant \Vdash_{P} \Longrightarrow N \cup \acute{N} \in \mathcal{N}$$

$$N \in \mathcal{N} \Leftrightarrow \hbar \llcorner N \in \mathcal{N}$$

$$\varnothing \in \mathcal{N} \ni \hbar$$

$$\mathcal{N} \ni \dot{N} \text{ disj } \Longrightarrow \Vdash_{P \cap \underbrace{N \cup \acute{N}}} = \Vdash_{P \cap N} + \Vdash_{P \cap \acute{N}} \underset{P = \varnothing}{\Longrightarrow} \Vdash \text{ fin add}$$

$$\begin{cases} \mathcal{N} \ni N_i \text{ disj} \\ N := \bigcup\limits_{i}^{\mathbb{N}} N_i \end{cases} \quad \Longrightarrow \bigcup_{i}^{n} N_i \in \mathcal{N} \Longrightarrow \Vdash_{P} = \Vdash_{P \cap \bigcup\limits_{i}^{n} N_i} + \Vdash_{P \ni \bigcup\limits_{i}^{n} N_i} = \sum_{i}^{n} \Vdash_{P \cap N_i} + \Vdash_{P \ni \bigcup\limits_{i}^{n} N_i} \geqslant \sum_{i}^{n} \Vdash_{P \cap N_i} + \Vdash_{P \llcorner N}$$

$$\underset{n \rightsquigarrow \infty}{\Longrightarrow} \Vdash_{P} \geqslant \sum_{i}^{\mathbb{N}} \Vdash_{P \cap N_i} + \Vdash_{P \llcorner N} \geqslant \Vdash_{P \cap N} + \Vdash_{P \llcorner N} \Longrightarrow N \in \mathcal{N} \sigma \text{ alg}$$

$$P = N : \nu_{N} \geqslant \sum_{i}^{\mathbb{N}} \nu_{N_i} \text{ meas}$$

$$M \subset N_0 \in \mathcal{N}$$

$$\nu_{N_0} = 0 \Longrightarrow \Vdash_{P} \geqslant \Vdash_{P \llcorner M} + \Vdash_{P \cap M} \Longrightarrow M \in \mathcal{N} \text{ voll}$$

$$\bar{\mathbb{R}}_{+} \underset{\neq}{\triangledown} \hbar = \frac{\bar{\mathbb{R}}_{+} \overset{\vdash}{\longleftarrow} \Re}{\bigcup\limits_{i}^{\mathbb{N}} R_i \in \Re \Longrightarrow \vdash_{\bigcup\limits_{i}^{\mathbb{N}} R_i} = \sum\limits_{i}^{\mathbb{N}} \vdash_{R_i} \text{ bed abz add}} \text{ semi-premeas}$$

$$\Vdash_P := \curlywedge_{P \subset \bigcup_i^{\mathbb{N}} R_i} \sum_i^{\mathbb{N}} \vdash_{R_i} \Rightarrow \Vdash \text{ out meas}$$

$$\Re \text{ meas } \Vdash|\Re = \vdash \ \Vdash_{\varnothing} = 0 \ \Vdash \text{ isoton}$$

$$\begin{cases} P_i \subset \hbar \\ \Vdash_{P_i} < \infty \end{cases} \Rightarrow \bigwedge_{\varepsilon}^{>0} \begin{cases} \bigvee P_i \subset \bigcup_j^{\mathbb{N}} R_i^j \\ \sum_j^{\mathbb{N}} \vdash_{R_i^j} \leqslant \Vdash_{P_i} + \varepsilon 2^{-i} \end{cases}$$

$$\Rightarrow \bigcup_i^{\mathbb{N}} P_i \subset \bigcup_i^{\mathbb{N}} \bigcup_j^{\mathbb{N}} R_i^j \Rightarrow \Vdash_{\bigcup_i^{\mathbb{N}} P_i} \leqslant \sum_i^{\mathbb{N}} \sum_j^{\mathbb{N}} \vdash_{R_i^j} \leqslant \varepsilon + \sum_i^{\mathbb{N}} \Vdash_{P_i} \underset{\varepsilon \rightsquigarrow 0}{\Rightarrow} \Vdash \text{ abz sub add}$$

$$R \in \Re$$

$$\begin{cases} P \subset \hbar \\ \Vdash_P < \infty \end{cases} \Rightarrow \bigwedge_{\varepsilon}^{>0} \bigwedge_{\bigcup_i^{\mathbb{N}} R_i \supset P} \Vdash_P + \varepsilon \geqslant \sum_i^{\mathbb{N}} \vdash_{R_i}$$

$$R_i \llcorner R = \bigcup_j^{N_i} R_i^j \Rightarrow R_i = \underbrace{R_i \cap R} \cup \bigcup_j^{N_i} R_i^j$$

$$\Rightarrow \Vdash_P + \varepsilon \geqslant \sum_i^{\mathbb{N}} \overbrace{\vdash_{R_i \cap R} + \sum_j^{N_i} \vdash_{R_i^j}} \geqslant \Vdash_{P \cap R} + \Vdash_{P \llcorner R} \underset{\varepsilon \rightsquigarrow 0}{\Rightarrow} R \text{ meas}$$

$$\Re \ni R \subset \bigcup_j^{\mathbb{N}} R_j$$

$$S_j = R_j \llcorner \overbrace{\bigcup_i^{j} R_i} = \bigcup_i^{N_j} R_j^i \Rightarrow R = \bigcup_j^{\mathbb{N}} R_j \cap R = \bigcup_j^{\mathbb{N}} S_j \cap R = \bigcup_j^{\mathbb{N}} \bigcup_i^{N_j} R_j^i \cap R$$

$$R_j = \bigcup_{k \leqslant j} S_k = \bigcup_{k \leqslant j} \bigcup_i^{N_k} R_k^i$$

$$\Rightarrow \sum_j^{\mathbb{N}} \vdash_{R_j} = \sum_j^{\mathbb{N}} \sum_k^{\leqslant j} \sum_i^{N_k} \vdash_{R_k^i} \geqslant \sum_j^{\mathbb{N}} \sum_i^{N_j} \vdash_{R_j^i} \geqslant \sum_j^{\mathbb{N}} \sum_i^{N_j} \vdash_{R_j^i \cap R} = \vdash_R \geqslant \Vdash_R \underset{\inf}{\Rightarrow} \Vdash_R = \vdash_R$$

$$\bar{\mathbb{R}}_+ \triangledown_{-0} \hbar = \frac{\bar{\mathbb{R}}_+ \overset{\vdash}{\leftarrow} \overset{\hbar}{\underset{0}{\triangle}}\tilde{} 2}{O \subset \acute{O} \Rightarrow \vdash_O \leqslant \vdash_{\acute{O}} : \quad O \Subset \hbar \Rightarrow \vdash_O < \infty : \quad \vdash_{O \cup \acute{O}} \geqslant \vdash_O + \vdash_{\acute{O}} : \quad \vdash_{\bigcup_i^{\mathbb{N}} O_i} \leqslant \sum_i^{\mathbb{N}} \vdash_{O_i} : \quad \vdash_O = \curlyvee_{\acute{O} \Subset O} \vdash_{\acute{O}}} \text{ top semi-pre}$$

$$\Rightarrow \begin{cases} \vdash_{\varnothing} = 0 \\ \vdash_{O \cup \acute{O}} = \vdash_{O} + \vdash_{\acute{O}} \end{cases}$$

$$\Vdash_P := \underset{P \subset O}{\overset{\vdash_O < \infty}{\curlywedge}} \vdash_O \Rightarrow \Vdash \text{ out meas}$$

$$\overset{\hbar}{\underset{0}{\triangle}} 2 \text{ meas } \Vdash\Big| \overset{\hbar}{\underset{0}{\triangle}} = \vdash$$

$$O \subset \acute{O} \Rightarrow \Vdash_O \leqslant \vdash_O \leqslant \vdash_{\acute{O}} \underset{\text{inf}}{\Rightarrow} \Vdash_O = \vdash_O$$

$$\Vdash_{\varnothing} = \vdash_{\varnothing} = 0$$

$$P \subset \acute{P} \subset \acute{O} \Rightarrow P \subset \acute{O} \Rightarrow \Vdash_P \leqslant \Vdash_{\acute{P}} \text{ isoton}$$

$$P_i \subset O_i \text{ off } \Rightarrow \bigcup_i^{\mathbb{N}} P_i \subset \bigcup_i^{\mathbb{N}} O_i \text{ off } \Rightarrow \Vdash_{\bigcup_i^{\mathbb{N}} P_i} \leqslant \vdash_{\bigcup_i^{\mathbb{N}} O_i} \leqslant \sum_i^{\mathbb{N}} \vdash_{O_i} \underset{\text{inf}}{\Rightarrow} \Vdash_{\bigcup_i^{\mathbb{N}} P_i} \leqslant \sum_i^{\mathbb{N}} \Vdash_{P_i} \text{ abz sub add}$$

$$\begin{cases} Q \sqsubset \hbar \supset O \supset P \\ \vdash_O < \infty \end{cases} \Rightarrow \bigwedge_{\varepsilon}^{>0} \bigvee_{\acute{O} \Subset O \llcorner Q} \vdash_{\acute{O}} > \vdash_{O \llcorner Q} - \varepsilon$$

$$O \llcorner \underbrace{O \llcorner \acute{O}^-} = \acute{O}^- \subset O \llcorner Q \Rightarrow O \cap Q \subset O \llcorner \acute{O}^-$$

$$\vdash_O + \varepsilon \geqslant \vdash_{\underbrace{O \llcorner \acute{O}^-} \cup \acute{O}} + \varepsilon = \vdash_{O \llcorner \acute{O}^-} + \vdash_{\acute{O}} + \varepsilon > \Vdash_{O \cap Q} + \Vdash_{O \llcorner Q} \underset{\varepsilon \rightsquigarrow 0}{\Rightarrow} \vdash_O \geqslant \Vdash_{O \cap Q} + \vdash_{O \llcorner Q}$$

$$\underset{\text{inf}}{\Rightarrow} \Vdash_P \geqslant \underset{O \supset P}{\overset{\vdash_O < \infty}{\curlywedge}} \left(\Vdash_{O \cap Q} + \vdash_{O \llcorner Q} \right) \geqslant \Vdash_{O \cap Q} + \Vdash_{O \llcorner Q} \Rightarrow Q \text{ meas } \Rightarrow \overset{\hbar}{\underset{0}{\triangle}} 2 \text{ meas}$$