

$$\ell \blacktriangleleft n = \{\ell \xrightarrow[\text{inj}]{} n\}$$

$$\begin{array}{ccc} \ell & \blacktriangleleft & n \\ & \downarrow \asymp & \\ & C(\ell) \times^{\ell} & \blacktriangleleft n \end{array}$$

$$\begin{array}{ccccc} & & \text{inj} & & \\ & & \pi \bowtie \nu & & \\ \ell & \xrightarrow[\pi]{\text{bij}} & \ell & \xrightarrow[\nu]{\text{str iso}} & n \end{array}$$

$$\ell \xrightarrow[\text{inj}]{\nu} n \Rightarrow \ell \text{ set } {}^\ell \mathfrak{U} = \{{}^0 \mathfrak{U} \dots {}^{\ell-1} \mathfrak{U}\} \subset n$$

$$\Rightarrow \bigvee_{\text{eind}} \pi \in C(\ell) \quad {}^{\pi^{-1}} {}^0 \mathfrak{U} < {}^{\pi^{-1}} {}^0 \mathfrak{U} < \dots < {}^{\pi^{-1}} {}^0 \mathfrak{U}$$

$${}^i \nu = {}^{\pi^{-1}} {}^i \mathfrak{U}$$

$$\sharp \ell \blacktriangleleft n = {}_\ell (n) = n(n-1)\cdots(n+1-\ell) \text{ falling power}$$

$$\text{LHS} = \underbrace{C(\ell)}_{\text{eind}} \underbrace{\ell \blacktriangleleft n}_{\text{def}} = \ell! \begin{bmatrix} n \\ \ell \end{bmatrix} = \text{RHS}$$