$$ {}^{N}\blacktriangle M = \{ N \xrightarrow[\text{surj}]{\daleth} M \} $$

$$ \sharp\, {}^{N}\blacktriangle M = \sum_{j}^{0|M} (-1)^{M-j} \begin{bmatrix} M \\ j \end{bmatrix} j^{\bar{N}} $$

$$ A = {}^{N}\triangle M : \quad \daleth \in {}^{N}\triangle M \Rightarrow \text{Eig}\, \daleth = {}^{N}\daleth \subset M $$

$$ S \subset M $$

$$ \underbrace{{}^{N}\triangle M}_{S}^{\subset} = \frac{\daleth \in {}^{N}\triangle M}{\text{Eig}\, \daleth \subset S} = \frac{\daleth \in {}^{N}\triangle M}{{}^{N}\daleth \subset S} = {}^{N}\triangle S \Rightarrow \sharp \underbrace{{}^{N}\triangle M}_{S}^{\subset} = \sharp\, {}^{N}\triangle S = |S|^{N} $$

$$ \underbrace{{}^{N}\triangle M}_{S}^{=} = \frac{\daleth \in {}^{N}\triangle M}{\text{Eig}\, \daleth = S} = \frac{\daleth \in {}^{N}\triangle M}{{}^{N}\daleth = S} = {}^{N}\blacktriangle S $$

$$ \underbrace{{}^{N}\triangle M}_{T}^{\subset} = \bigcup_{S \subset T} \underbrace{{}^{N}\triangle M}_{S}^{=} \underset{\text{Sieb}}{\Longrightarrow} \sharp \underbrace{{}^{N}\triangle M}_{T}^{=} = \sum_{S \subset T} (-1)^{T \llcorner S} \sharp \underbrace{{}^{N}\triangle M}_{S}^{\subset} $$

$$ \sharp\, {}^{N}\blacktriangle M = \sharp \underbrace{{}^{N}\triangle M}_{M}^{=} = \sum_{S} (-1)^{M \llcorner S} \sharp \underbrace{{}^{N}\triangle M}_{S}^{\subset} = \sum_{S} (-1)^{M \llcorner S} |S|^{N} = \sum_{j}^{0|M} (-1)^{M-j} j^{N} \sum_{|S| = j} 1 = \text{RHS} $$