$$\text{order } \hbar : P \in \triangle$$

$$\overset{\ell}{\triangle}\hbar = \frac{\ell \xrightarrow{\iota} \hbar \text{ strong isoton}}{{}^{0}\iota < {}^{1}\iota < \cdots < {}^{\ell-1}\iota} = \underbrace{\overset{\ell}{\triangle}\hbar}_{\text{isoton}} \cap \underbrace{\overset{\ell}{\blacktriangle}\hbar}_{\text{inj}}$$

$$1 \leqslant \ell \Rightarrow {}_x\overbrace{\underbrace{P-I}}^{\ell}{}^{y} = \sharp\, {}^{\ell-1}\triangle x|y$$

$${}_x\underbrace{P-I}^{y} = {}_xP^y - {}_xI^y = \begin{cases} 0 & x = y \\ 1 & x < y \end{cases} = \sharp\, {}^{0}\triangle x|y$$

$${}_x\underbrace{\overbrace{P-I} * \overbrace{P-I}}^{y} = \sum_{x \leqslant z \leqslant y} {}_x\underbrace{P-I}^{z}\ {}_z\underbrace{P-I}^{y} = \sum_{x < z < y} 1 = \sharp\, {}^{1}\triangle x|y$$

$${}_x\overbrace{P-I}^{\ell}{}^{y} = \sum_{x = z_0 < z_1 < \cdots < z_{\ell-1} < z_\ell = y} 1 = \sharp\, {}^{\ell-1}\triangle x|y$$

$${}_x\overbrace{\underbrace{1+t}I - tP}^{-1}{}^{y} = \sum_{1 \leqslant \ell} t^\ell \sharp\, {}^{\ell-1}\triangle x|y$$

$$\overbrace{\underbrace{1+t}I - tP}^{-1} = \overbrace{I - t\underbrace{P-I}}^{-1} = \sum_{1 \leqslant \ell} \overbrace{t\underbrace{P-I}}^{\ell} = \sum_{1 \leqslant \ell} t^\ell\ \overbrace{P-I}^{\ell} \Rightarrow \text{LHS} = \sum_{1 \leqslant \ell} t^\ell\ {}_x\overbrace{P-I}^{\ell}{}^{y} = \text{RHS}$$

$$\text{Moebius } {}_x\overset{-1}{P}{}^{y} = \sum_{1 \leqslant \ell} (-1)^\ell \sharp\, {}^{\ell-1}\triangle x|y$$

$$t = -1$$