

$$\text{order } \mathfrak{h}: P \in \mathbb{N}$$

$$\ell \triangleleft \mathfrak{h} = \frac{\ell \xrightarrow{\mathfrak{h}} \mathfrak{h} \text{ weak isoton}}{^0\mathfrak{h} \leqslant ^1\mathfrak{h} \leqslant \dots \leqslant ^{\ell-1}\mathfrak{h}}$$

$$\underbrace{{}^{1+\ell}P}_x^y = \#^\ell \triangleleft x|y$$

$${}_x\dot{P}^y = {}_xP^y = {}_{x\leqslant y}\#^0 \triangleleft x|y$$

$${}_x\underbrace{P\ast P}^y = \sum_{x\leqslant z\leqslant y} \underbrace{{}_xP^z}_{=1} \underbrace{{}_zP^y}_{=1} = \sum_z^{x|y} 1 = \#x|y = \#^1 \triangleleft x|y$$

$${}_x\underbrace{P\ast P\ast P}^y = \sum_{x\leqslant z_1\leqslant z_2\leqslant y} 1 = \#^2 \triangleleft x|y$$

$${}_x\underbrace{{}^{1+\ell}P}_x^y = \sum_{x\leqslant z_1\leqslant \dots \leqslant z_\ell\leqslant y} 1 = \#^\ell \triangleleft x|y$$

$${}_x\underbrace{\overline{P\dot{I}-tP}}^{y}_{-1} = \sum_{0\leqslant \ell} t^\ell \#^\ell \triangleleft x|y$$

$$P\widehat{\overline{I-tP}}^{-1} = P\sum_{0\leqslant \ell} \widehat{tP}^\ell = \sum_{0\leqslant \ell} t^\ell \dot{P}^{1+\ell} \Rightarrow \text{LHS} = \sum_{0\leqslant \ell} t^\ell \underbrace{{}^{1+\ell}P}_x^y = \text{RHS}$$