

$$\begin{array}{ccc} \mathbb{h}_{\mathbb{C}} & \xleftarrow{\quad} & \mathbb{C} \\ \mathbb{L}' & & \end{array}$$

$$\begin{array}{c} \mathbb{h}_{\mathbb{C}}^{\mathbb{N}} \ni \mathbb{L}^J = \sum_{j \in J} \mathbb{L}^j \text{ dual standard basis} \end{array}$$

$$\mathbb{L}^I \star \mathbb{L}^J = \mathbb{L}^I \circ \mathbb{L}^J = \det \mathbb{L}^i \star \mathbb{L}^j = \det_i \eta^j = {}_I \eta^J = {}_I \circ \eta^J$$

$$\star \mathbb{L} = \mathbb{L}^I \eta^I$$

$$\mathbb{L}^I = (\star \mathbb{L})_I \eta^I$$

$$*\mathbb{L}^I = \mathbb{L}^{N \sqcup I} \xrightarrow{I > N \sqcup I} {}_I \eta^I$$

$$\mathbb{L} = \mathbb{L} \underbrace{\mathbb{L}' \mathbb{L}}$$

$${}_M \mathbb{L} \mathbb{L}^N = \det (\mu \mathbb{L} \mathbb{L}^\nu) = \det \mu \delta^\nu = {}_M \delta^N$$

$$\mathbb{H} = \mathbb{L} \underbrace{\mathbb{L}' \mathbb{H}}$$

$$\mathbb{L}^I \mathbb{L}^J = \det \mathbb{L}^i \mathbb{L}^j = \det_i \delta^j = {}_I \delta^J = {}_I \mathbb{L} \mathbb{L}^J$$

$$\mathbb{L}^I = {}_I \mathbb{L}$$

$$\begin{array}{ccccc} \mathbb{h}_{\mathbb{C}}^{\mathbb{N}} & \xleftarrow{\quad} & & \xleftarrow{\quad} & \mathbb{C} \\ \mathbb{L}' & & \mathbb{L}' & & \mathbb{L}' \\ \downarrow & & \downarrow & & \downarrow \\ \mathbb{h}_{\mathbb{C}}^{\mathbb{N}} & \xleftarrow{\quad} & & \xleftarrow{\quad} & \mathbb{C} \end{array}$$

$$\begin{array}{c} \mathbb{L}^I \star_h \mathbb{L}^J = \begin{cases} \mathbb{L}^I \mathbb{h} \mathbb{L}^J & = {}^h \mathbb{h}^{IJ} \\ \mathbb{L}^I \mathbb{V}_h \mathbb{L}^J & = {}_I \mathbb{L} \mathbb{V}_h \mathbb{L}^J = \det \mathbb{L}^i \star_h \mathbb{L}^j = \det {}_I \mathbb{V}_h^J = {}_I \mathbb{V}_h^J \end{cases} \end{array}$$

$$\begin{array}{c} \mathbb{z} \mathbb{L} = \sum_{|I|=|J|} \mathbb{L}^I \mathbb{z} \mathbb{L}^J \end{array}$$

$$\begin{array}{c} \mathbb{L}^J = \sum_{|I|=|J|} (\mathbb{z} {}_I \mathbb{L}) {}_I \mathbb{V}_z^J \end{array}$$

$${}_M \mathbf{L} \sum_I \mathbf{L}^I {}_I \tilde{\mathbf{A}}^J = \sum_I {}_M \delta^I {}_I \tilde{\mathbf{A}}^J = {}_M \tilde{\mathbf{A}}^J = {}_M \mathbf{L} \tilde{\mathbf{x}} {}_J \mathbf{L}$$

$${}_M \mathbf{L} \mathbf{L}^J = {}_M \delta^J = \det_M \left( \nabla_z {}^z \mathbf{A} \right)^J = \sum_{|I|=|J|} {}_M \nabla_z^I {}_I {}^z \mathbf{A}^J = \sum_I ({}_M \mathbf{L} \mathbf{x} {}_I \mathbf{L}) {}^z \mathbf{A}^J$$

$$\tilde{\mathbf{x}} {}_I \nabla = \mathbf{A}^I {}_I \eta^I$$

$$\mathbf{x} \mathbf{A}^I = (\nabla) {}_I \eta^I$$

$$\begin{aligned} \overline{\mathbf{h}} \overline{\mathbf{L}} \overline{\mathbb{C}} &\ni \begin{cases} {}^h \mathbf{A}^J & = {}^h \mathbf{A}_9 \mathbf{L} \\ \mathbf{A}^J & = \underset{j \in J}{\mathbf{x}} \mathbf{A}^j = \mathbf{A} \mathbf{L}^J \end{cases} \text{ dual ONBasis} \end{aligned}$$

$$\begin{cases} {}^h \mathbf{A} & = {}^h \mathbf{A}_I \mathbf{L} \\ \mathbf{A} & = \mathbf{A}_I \mathbf{L} \end{cases}$$

$$\begin{aligned} \begin{aligned} {}^h \mathbf{A}_I \mathbf{x} {}^h \mathbf{A}^J &= {}^h \mathbf{A}_I {}^h \mathbf{A}^J = \mathbf{A}_I \underbrace{{}^h \mathbf{A}_{\eta} {}^h \mathbf{A}}_{\eta} {}^h \mathbf{A}^J = \underbrace{{}^h \mathbf{A}_I {}^h \mathbf{A}}_{\eta} \underbrace{{}^h \mathbf{A}^J}_{\mathbf{A}} \\ \mathbf{A}^I \mathbf{x} \mathbf{A}^J &= \det \mathbf{A}^i \left( \nabla \eta \nabla \right) \mathbf{A}^j = \det \left( \nabla \mathbf{A}^i \right) \eta \left( \nabla \mathbf{A}^j \right) = \det {}_i \nabla \eta {}_i \mathbf{L}^j = \mathbf{A}^I \nabla_h \mathbf{A}^J = \mathbf{A}^I \underbrace{{}^* \eta \nabla}_{\mathbf{A}} \mathbf{A}^J = \underbrace{{}^* \nabla \eta \nabla}_{\mathbf{A}} \mathbf{A}^J = \mathbf{A}^I \underbrace{{}^* \nabla}_{\mathbf{A}} \mathbf{A}^J \end{aligned} &= \underbrace{{}^h \mathbf{A} \mathbf{L}^I} \underbrace{{}^h \mathbf{A}^J}_{\mathbf{A}} \end{aligned}$$

$$\mathbf{*}_z \mathbf{A}^I = \mathbf{A}^{N \sqcup I} \overline{I > \mathcal{O}^{N \sqcup I}} {}_I \eta^I$$

$$\mathbf{*}_z \mathbf{L}^J = \sum_{|I|=|J|} \mathbf{L}^{N \sqcup I} \overline{I > \mathcal{O}^{N \sqcup I}} {}_I \nabla_z^J \left( {}_N \eta^N / {}_N \nabla_z^N \right)^{1/2}$$

$$\mathbf{A}^N = c \mathbf{L}^N$$

$${}_N \eta^N = \mathbf{A}^N \mathbf{x} {}_z \mathbf{A}^N = c^2 \mathbf{L}^N \mathbf{x} \mathbf{L}^N = c^2 {}_N \nabla_z^N \Rightarrow \text{LHS} = \sum_I {}_I \mathbf{L} \models \mathbf{A}^N {}_I \nabla_z^J = \sum_I {}_I \mathbf{L} \models \mathbf{L}^N \left( {}_N \eta^N / {}_N \nabla_z^N \right)^{1/2} {}_I \nabla_z^J = \text{RHS}$$

$$\mathbf{H} = \begin{cases} {}^h \mathbf{A}' \mathbf{A}' \mathbf{H} \\ \nabla \mathbf{A}' \mathbf{A}' \mathbf{H} \end{cases} \quad {}_I \delta^J = \begin{cases} {}^h \mathbf{A}' \mathbf{A}' \mathbf{A}^J \\ {}_I \nabla \mathbf{A}' \mathbf{A}^J \end{cases}$$

$$\mathbf{A}' = \begin{cases} {}^h \mathbf{A}' \mathbf{A}' \mathbf{A} \\ \mathbf{A}' \mathbf{A}' \mathbf{A} \end{cases}$$

$${}_M \delta^N = \begin{cases} {}^h \mathbf{A}' \mathbf{A}' \mathbf{A}^N \\ {}_M \mathbf{A}' \mathbf{A}^N \end{cases}$$

$$\mathbf{L}' \mathbf{H} = \begin{cases} {}^h \mathbf{A}' \mathbf{A}' \mathbf{H} \\ \nabla \mathbf{A}' \mathbf{H} \end{cases} = \begin{cases} {}^h \mathbf{A}' \mathbf{A}' \mathbf{H} \\ \mathbf{A}' \mathbf{H} \end{cases}$$

$$\mathcal{L}^J = \begin{cases} {}^h\mathfrak{z}_L {}^h\mathfrak{z}_L^J & = {}^h\mathfrak{z}_L^{\circ L} {}^h\mathfrak{z}_L^J \\ {}^h\mathfrak{z}_L^J & = {}^h\mathfrak{z}_L^L {}^h\mathfrak{z}_L^J \end{cases}$$

$$\mathcal{L}'_1 = \begin{cases} {}^h\mathfrak{z}_1 {}^h\mathfrak{z}_1^{\circ}, 1 & = {}^h\mathfrak{z}_1' {}^h\mathfrak{z}_1^{\circ}, 1 \\ {}^h\mathfrak{z}_1^{\circ}, 1 & = {}^h\mathfrak{z}_1' {}^h\mathfrak{z}_1^{\circ}, 1 \end{cases}$$

$$\mathcal{L}^N = \begin{cases} {}^h\mathfrak{z}_N {}^h\mathfrak{z}_N^{\circ} & = {}^h\mathfrak{z}_N^K {}^h\mathfrak{z}_N^K \\ {}^h\mathfrak{z}_N^{\circ} & = {}^h\mathfrak{z}_N^K {}^h\mathfrak{z}_N^K \end{cases}$$

$$\begin{cases} {}^h\mathfrak{z}_1' \mathfrak{H} & = \mathcal{L}'_1 {}^h\mathfrak{z}_1' \mathfrak{H} = {}^h\mathfrak{z}_1 \mathcal{L}'_1 \mathfrak{H} \\ {}^h\mathfrak{z}_1' \mathfrak{H} & = \mathcal{L}'_1 {}^h\mathfrak{z}_1' \mathfrak{H} = {}^h\mathfrak{z}_1 \mathcal{L}'_1 \mathfrak{H} \end{cases}$$

$$\begin{cases} {}^h\mathfrak{z}_L^J & = \mathcal{L}^L {}^h\mathfrak{z}_L^J = {}^h\mathfrak{z}_L \mathcal{L}^J \\ {}^h\mathfrak{z}_L^J & = \mathcal{L}^L {}^h\mathfrak{z}_L^J = {}^h\mathfrak{z}_L \mathcal{L}^J \end{cases}$$

$$\begin{cases} {}^h\mathfrak{z}_1^{\circ}, 1 & = \mathcal{L}'_1 {}^h\mathfrak{z}_1^{\circ}, 1 = {}^h\mathfrak{z}_1 \mathcal{L}'_1, 1 \\ {}^h\mathfrak{z}_1^{\circ}, 1 & = \mathcal{L}'_1 {}^h\mathfrak{z}_1^{\circ}, 1 = {}^h\mathfrak{z}_1 \mathcal{L}'_1, 1 \end{cases}$$

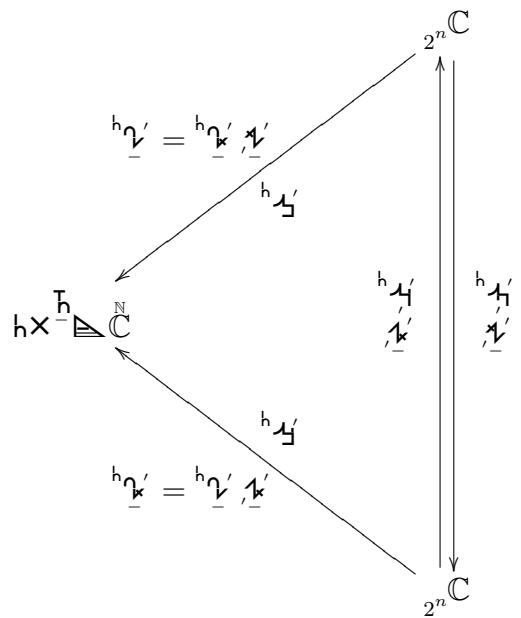
$$\begin{cases} {}^h\mathfrak{z}_N^{\circ N} & = \mathcal{L}^K {}^h\mathfrak{z}_N^{\circ N} = {}^h\mathfrak{z}_N \mathcal{L}^N \\ {}^h\mathfrak{z}_N^{\circ N} & = \mathcal{L}^K {}^h\mathfrak{z}_N^{\circ N} = {}^h\mathfrak{z}_N \mathcal{L}^N \end{cases}$$

$$\begin{cases} {}^h\mathfrak{z}_1' \mathfrak{H} & = \mathcal{L}'_1 {}^h\mathfrak{z}_1' \mathfrak{H} = {}^h\mathfrak{z}_1 \mathcal{L}'_1 \mathfrak{H} \\ {}^h\mathfrak{z}_1' \mathfrak{H} & = \mathcal{L}'_1 {}^h\mathfrak{z}_1' \mathfrak{H} = {}^h\mathfrak{z}_1 \mathcal{L}'_1 \mathfrak{H} \end{cases}$$

$$\begin{cases} {}^h\mathfrak{z}_M^J & = {}^h\mathfrak{z}_M^L {}^h\mathfrak{z}_L^J = {}^h\mathfrak{z}_M \mathcal{L}^J \\ {}^h\mathfrak{z}_M^J & = {}^h\mathfrak{z}_M^L {}^h\mathfrak{z}_L^J = {}^h\mathfrak{z}_M \mathcal{L}^J \end{cases}$$

$$\begin{cases} {}^h\mathfrak{z}_1^{\circ}, 1 & = \mathcal{L}'_1 {}^h\mathfrak{z}_1^{\circ}, 1 = {}^h\mathfrak{z}_1 \mathcal{L}'_1, 1 \\ {}^h\mathfrak{z}_1^{\circ}, 1 & = \mathcal{L}'_1 {}^h\mathfrak{z}_1^{\circ}, 1 = {}^h\mathfrak{z}_1 \mathcal{L}'_1, 1 \end{cases}$$

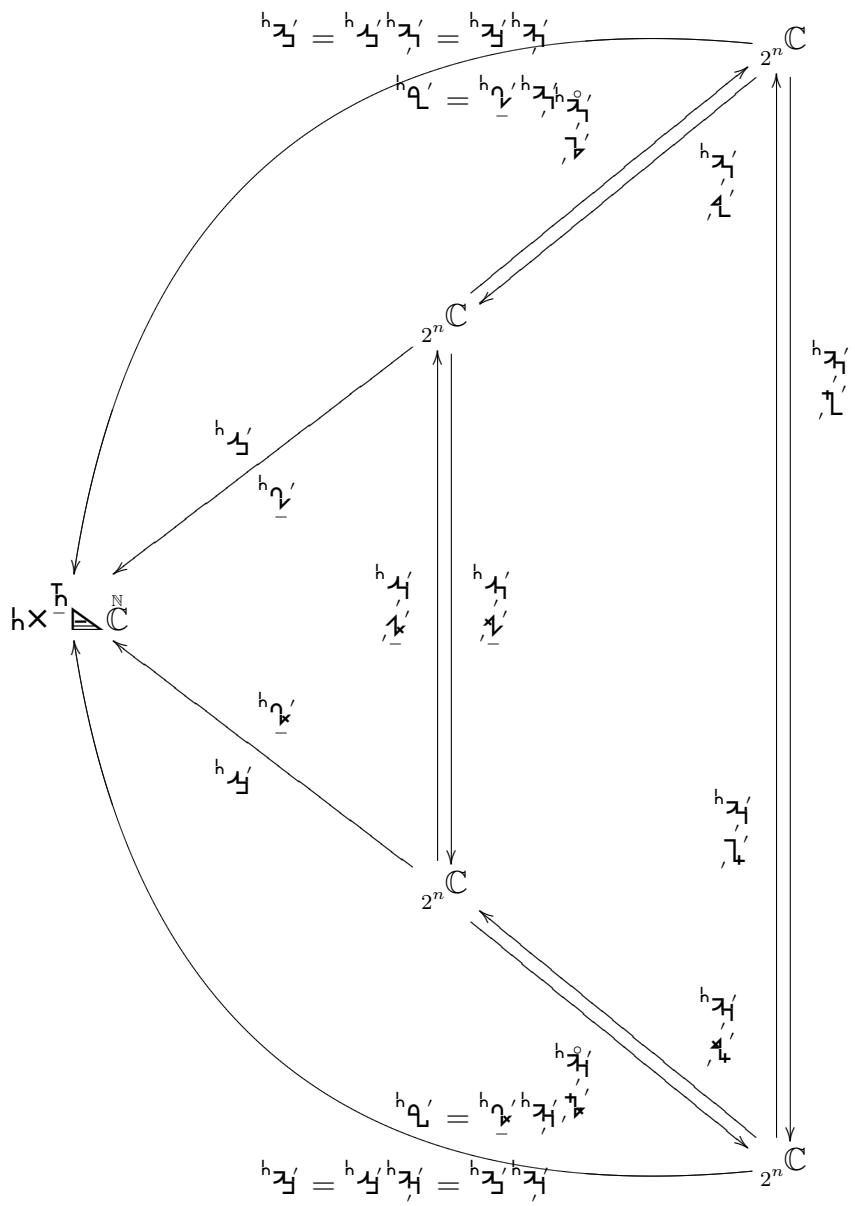
$$\begin{cases} {}^h\mathfrak{z}_I^{\circ N} & = {}^h\mathfrak{z}_I^L {}^h\mathfrak{z}_L^N = {}^h\mathfrak{z}_I \mathcal{L}^N \\ {}^h\mathfrak{z}_I^{\circ N} & = {}^h\mathfrak{z}_I^L {}^h\mathfrak{z}_L^N = {}^h\mathfrak{z}_I \mathcal{L}^N \end{cases}$$



$h \times \overset{h}{\Delta} \overset{\mathbb{N}}{\mathbb{C}} \ni \underset{\gamma'}{\gamma}$  holonomic basis

$$\gamma = \underbrace{\mathcal{U}_h}_{h \gamma'} \gamma'$$

$$_M \delta^N = {}_{_M \mathcal{U}_h} \overset{h \gamma'}{\gamma} {}^N$$



$$h \times \overline{h} \ominus \mathbb{C} \ni \begin{cases} {}^h \mathfrak{z}^J \\ {}^h \mathfrak{q}^J \end{cases} = \sum_{j \in J} {}^h \mathfrak{q}^j \text{ dual ONbasis}$$

$${}^h \mathfrak{q}^I \boxtimes {}^h \mathfrak{q}^J = {}_I \eta^J$$

$$\star {}_I \mathfrak{b}_h = {}^h \mathfrak{q}^I {}_I \eta^I$$

$${}^h \mathfrak{q}^I = (\star {}_I \mathfrak{b}_h) {}_I \eta^I$$

$$*\underline{q}^I = \underline{q}^{N+I} \overline{I > N+I}_I \eta^I$$

$$\underline{\nu} = \begin{cases} \underline{\pi}^{\underline{h}\underline{q}'\underline{h}} \\ \underline{\pi}_{\underline{h}}^{\underline{h}\underline{q}'\underline{h}} \end{cases}$$

$$_I \delta^J = \begin{cases} ^h\underline{\pi}^h \underline{q}^J \\ _I \underline{\pi}_{\underline{h}}^h \underline{q}^J \end{cases}$$

$$\begin{cases} ^h\underline{q}'\underline{h} \\ ^h\underline{q}'\underline{h} \end{cases} = \begin{cases} ^h\underline{\nu}^h \underline{q}'\underline{h} \\ ^h\underline{\nu}^h \underline{q}'\underline{h} \end{cases} \quad \begin{cases} ^h\underline{q}^J \\ ^h\underline{q}^J \end{cases} = \begin{cases} \underline{\nu}^L h^L \underline{q}^J \\ ^h\underline{\nu}^L \underline{q}^J \end{cases}$$

$$^h\underline{\nu}', \underline{1} = \begin{cases} ^h\underline{\pi}^h \underline{\nu}', \underline{1} \\ ^h\underline{q}' \underline{\pi}', \underline{1} \end{cases}$$

$$^h\underline{\nu}^N = \begin{cases} ^h\underline{\pi}^K h^N \underline{\pi}^N \\ ^h\underline{q}^K \underline{\pi}^N \end{cases}$$

$$\begin{cases} ^h\underline{q}'\underline{h} \\ ^h\underline{q}'\underline{h} \end{cases} = \begin{cases} \underline{\nu}_h^h \underline{q}'\underline{h} \\ \underline{\nu}_h^h \underline{q}'\underline{h} \end{cases} \quad \begin{cases} ^h\underline{q}^J \\ _M \underline{q}^J \end{cases} = \begin{cases} \underline{\nu}_h^h \underline{q}^J \\ _M \underline{\nu}_h^h \underline{q}^J \end{cases}$$

$$\begin{cases} ^h\underline{\pi}', \underline{1} \\ ^h\underline{\pi}', \underline{1} \end{cases} = \begin{cases} ^h\underline{\pi}^h \underline{\nu}', \underline{1} \\ ^h\underline{\pi}^h \underline{\nu}', \underline{1} \end{cases}$$

$$\begin{cases} ^h\underline{\pi}^N \\ _I \underline{\pi}^N \end{cases} = \begin{cases} ^h\underline{\pi}^h \underline{\nu}^N \\ ^h\underline{\pi}^h \underline{\nu}^N \end{cases}$$

