

$$\begin{array}{ccc} \mathbb{H} & \triangleleft & \mathbb{C} \\ \mathbb{L}' & \longleftarrow & 2^n \end{array}$$

$$\begin{array}{c} \mathbb{H} \\ \triangleleft \\ \mathbb{J} \end{array} \ni \mathbb{L}^J = \sum_{j \in J} \mathbb{L}^j \quad \text{dual standard basis}$$

$$\mathbb{L}^I \star \mathbb{L}^J = \mathbb{L}^I \circ \mathbb{L}^J = \det \mathbb{L}^i \star \mathbb{L}^j = \det_i \eta^j = {}_I \eta^J = {}_I \circ \eta^J$$

$$\star \mathbb{L} = \mathbb{L}^I \eta^I$$

$$\mathbb{L}^I = (\star \mathbb{L})_I \eta^I$$

$$*\mathbb{L}^I = \mathbb{L}^{N \sqcup I} \xrightarrow{I > N \sqcup I} \mathbb{L}^I$$

$$\mathbb{A} = \mathbb{L} \underbrace{\mathbb{L}' \mathbb{A}}_M \mathbb{L} \mathbb{L}^N = \det (\mu \mathbb{L} \nu) = \det \mu \delta^\nu = {}_M \delta^N$$

$$\mathbb{H} = \mathbb{L} \underbrace{\mathbb{L}' \mathbb{H}}_I$$

$$\mathbb{L}^I \mathbb{L}^J = \det \mathbb{L}^i \mathbb{L}^j = \det_i \delta^j = {}_I \delta^J = {}_I \mathbb{L} \mathbb{L}^J$$

$$\mathbb{L}^I = {}_I \mathbb{L}$$

$$\begin{array}{ccc} \mathbb{H} & \triangleleft & \mathbb{C} \\ \mathbb{L}' & \nearrow & \downarrow \\ \mathbb{L} & & \mathbb{L}' \\ \mathbb{H} & \searrow & \downarrow \\ & & \mathbb{C} \end{array}$$

$$\mathbb{L}^I \star \mathbb{L}^J = \begin{cases} \mathbb{L}^I \mathbb{H} \mathbb{L}^J & = \mathbb{H} \mathbb{L}^J \\ \mathbb{L}^I \mathbb{V}_h \mathbb{L}^J & = {}_I \mathbb{V}_h \mathbb{L}^J = \det \mathbb{L}^i \star \mathbb{L}^j = \det_i \mathbb{V}_h^j = {}_I \mathbb{V}_h^J \end{cases}$$

$$\mathbf{x}^z \mathbb{L} = \sum_{|I|=|J|} \mathbb{L}^I \mathbf{x}^z \mathbb{L}^J$$

$$\mathbb{L}^J = \sum_{|I|=|J|} \left(\mathbf{x}^z {}_I \mathbb{L} \right) {}_I \mathbb{V}_z^J$$

$${}_M \mathbb{L} \sum_I \mathbb{L}^I {}_I \tilde{\mathbb{A}}^J = \sum_I {}_M \delta^I {}_I \tilde{\mathbb{A}}^J = {}_M \tilde{\mathbb{A}}^J = {}_M \mathbb{L} \mathbf{x}^z \mathbb{L}$$

$${}_M\mathbf{L} \cdot \mathbf{L}^J = {}_M\delta^J = \det_M \left(\nabla_z {}^z \mathbb{L} \right)^J = \sum_{|I|=|J|} {}_M \nabla_z {}^I {}^z \mathbb{L}^J = \sum_I ({}_M\mathbf{L} \times {}_I\mathbf{L}) {}^z \mathbb{L}^J$$

$$\tilde{\mathbf{x}}_I \nabla = \mathbf{A}_I^I \eta^I$$

$$\mathbf{x} \mathbf{A}^I = (\nabla_I) {}_I \eta^I$$

$$\begin{aligned} \mathbf{h}_{\mathbb{L}^N} &\ni \begin{cases} {}^h \mathbf{A}^J & = {}^h \mathbf{A}_9 \mathbf{L} \\ \mathbf{A}^J & = \sum_j \mathbf{A}^j = \mathbf{A} \mathbf{L}^J \end{cases} \text{ dual ONBasis} \end{aligned}$$

$$\begin{cases} {}^h \mathbf{A} & = {}^h \mathbf{A}_I \mathbf{L} \\ \mathbf{A} & = \mathbf{A}_I \mathbf{L} \end{cases}$$

$$\begin{aligned} \begin{cases} {}^h \mathbf{A}_I \underset{h}{\mathbf{x}} {}^h \mathbf{A}^J & = {}^h \mathbf{A}_I {}^h \mathbf{A}^J = \underbrace{{}^h \mathbf{A}_I \eta \mathbf{A}^J}_{\mathbf{A}^I \mathbf{x} \mathbf{A}^J} = \underbrace{{}^h \mathbf{A}_I \eta \mathbf{A}^J}_{\det \mathbf{A}^I \left(\nabla \eta \nabla \right) \mathbf{A}^J} = \det \left(\nabla \mathbf{A}^I \right) \eta \left(\nabla \mathbf{A}^J \right) = \det {}_I \nabla \mathbf{L}^J = \mathbf{A}_I^I \nabla_h \mathbf{A}^J = \mathbf{A}_I^I \underbrace{\nabla_h \mathbf{A}^J}_{\mathbf{A}^I \mathbf{x} \mathbf{A}^J} = \underbrace{\nabla_h \mathbf{A}^I}_{\mathbf{A}^I \mathbf{x} \mathbf{A}^J} \underbrace{\mathbf{A}^J}_{\mathbf{A}^I \mathbf{x} \mathbf{A}^J} = \underbrace{\mathbf{A}^I}_{\mathbf{A}^I \mathbf{x} \mathbf{A}^J} \underbrace{\mathbf{A}^J}_{\mathbf{A}^I \mathbf{x} \mathbf{A}^J} \end{cases} \\ \mathbf{x}_z \mathbf{A}^I = \mathbf{A}^{N \sqcup I} \xrightarrow{I > N \sqcup I} {}_I \eta^I \end{aligned}$$

$$\mathbf{x}_z \mathbf{L}^J = \sum_{|I|=|J|} \mathbf{L}^{N \sqcup I} \xrightarrow{I > N \sqcup I} {}_I \nabla_z \left({}_N \eta^N / {}_N \nabla_z \right)^{1/2}$$

$$\mathbf{A}^N = c \mathbf{L}^N$$

$${}_N \eta^N = \mathbf{A}^N \mathbf{x}_z \mathbf{A}^N = c^2 \mathbf{L}^N \mathbf{x} \mathbf{L}^N = c^2 {}_N \nabla_z {}_N \Rightarrow \text{LHS} = \sum_I {}_I \mathbf{L} \models \mathbf{A}^N {}_I \nabla_z {}_I \mathbf{L}^J = \sum_I {}_I \mathbf{L} \models \mathbf{L}^N \left({}_N \eta^N / {}_N \nabla_z {}_N \right)^{1/2} {}_I \nabla_z {}_I \mathbf{L}^J = \text{RHS}$$

$$\mathcal{H} = \begin{cases} {}^h \mathbf{A} & \text{, } \\ \nabla \mathbf{A} & \text{, } \end{cases}$$

$$I \delta^J = \begin{cases} {}^h \mathbf{A}_I {}^h \mathbf{A}^J \\ \nabla_I \mathbf{A}^J \end{cases}$$

$$\mathbf{A} = \begin{cases} {}^h \mathbf{A} & \text{, } \\ \nabla \mathbf{A} & \text{, } \end{cases}$$

$$M \delta^N = \begin{cases} {}^h \mathbf{A}_M {}^h \mathbf{A}^N \\ \nabla_M \mathbf{A}^N \end{cases}$$

$$\mathbf{L}' \cdot \mathcal{H} = \begin{cases} {}^h \mathbf{A}_I {}^h \mathbf{A}' \mathcal{H} & = {}^h \mathbf{A}' \mathbf{A}' \mathcal{H} \\ \nabla_I \mathbf{A}' \mathcal{H} & = \nabla' \mathbf{A}' \mathcal{H} \end{cases}$$

$$\mathcal{L}^J = \begin{cases} {}^h\mathfrak{z}_L {}^h\mathfrak{z}_L^J & = {}^h\mathfrak{z}_L^{\circ L} {}^h\mathfrak{z}_L^J \\ {}^h\mathfrak{z}_L^J & = {}^h\mathfrak{z}_L^L {}^h\mathfrak{z}_L^J \end{cases}$$

$$\mathcal{L}'_1 = \begin{cases} {}^h\mathfrak{z}_1 {}^h\mathfrak{z}_1^{\circ}, 1 & = {}^h\mathfrak{z}_1' {}^h\mathfrak{z}_1^{\circ}, 1 \\ {}^h\mathfrak{z}_1^{\circ}, 1 & = {}^h\mathfrak{z}_1' {}^h\mathfrak{z}_1^{\circ}, 1 \end{cases}$$

$$\mathcal{L}^N = \begin{cases} {}^h\mathfrak{z}_N {}^h\mathfrak{z}_N^{\circ} & = {}^h\mathfrak{z}_N^K {}^h\mathfrak{z}_N^K \\ {}^h\mathfrak{z}_N^{\circ} & = {}^h\mathfrak{z}_N^K {}^h\mathfrak{z}_N^K \end{cases}$$

$$\begin{cases} {}^h\mathfrak{z}_1' \mathfrak{H} & = \mathcal{L}'_1 {}^h\mathfrak{z}_1' \mathfrak{H} = {}^h\mathfrak{z}_1 \mathcal{L}'_1 \mathfrak{H} \\ {}^h\mathfrak{z}_1' \mathfrak{H} & = \mathcal{L}'_1 {}^h\mathfrak{z}_1' \mathfrak{H} = {}^h\mathfrak{z}_1 \mathcal{L}'_1 \mathfrak{H} \end{cases}$$

$$\begin{cases} {}^h\mathfrak{z}_L^J & = \mathcal{L}^L {}^h\mathfrak{z}_L^J = {}^h\mathfrak{z}_L \mathcal{L}^J \\ {}^h\mathfrak{z}_L^J & = \mathcal{L}^L {}^h\mathfrak{z}_L^J = {}^h\mathfrak{z}_L \mathcal{L}^J \end{cases}$$

$$\begin{cases} {}^h\mathfrak{z}_1', 1 & = \mathcal{L}'_1 {}^h\mathfrak{z}_1', 1 = {}^h\mathfrak{z}_1 \mathcal{L}'_1, 1 \\ {}^h\mathfrak{z}_1', 1 & = \mathcal{L}'_1 {}^h\mathfrak{z}_1', 1 = {}^h\mathfrak{z}_1 \mathcal{L}'_1, 1 \end{cases}$$

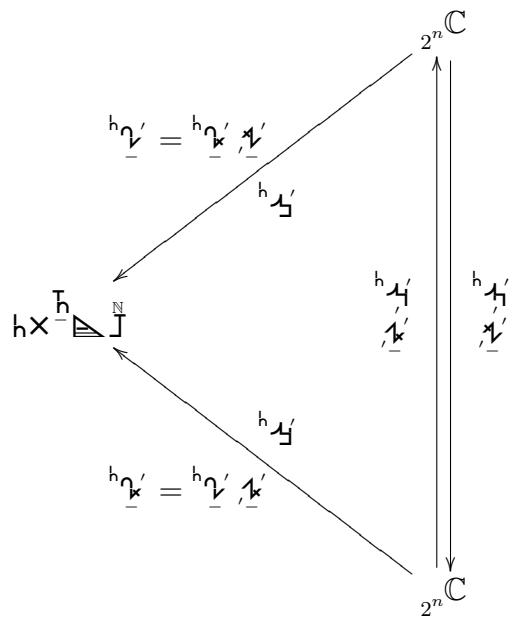
$$\begin{cases} {}^h\mathfrak{z}_N^{\circ N} & = \mathcal{L}^K {}^h\mathfrak{z}_N^{\circ N} = {}^h\mathfrak{z}_N \mathcal{L}^N \\ {}^h\mathfrak{z}_N^{\circ N} & = \mathcal{L}^K {}^h\mathfrak{z}_N^{\circ N} = {}^h\mathfrak{z}_N \mathcal{L}^N \end{cases}$$

$$\begin{cases} {}^h\mathfrak{z}_1' \mathfrak{H} & = \mathcal{L}'_1 {}^h\mathfrak{z}_1' \mathfrak{H} = {}^h\mathfrak{z}_1 \mathcal{L}'_1 \mathfrak{H} \\ {}^h\mathfrak{z}_1' \mathfrak{H} & = \mathcal{L}'_1 {}^h\mathfrak{z}_1' \mathfrak{H} = {}^h\mathfrak{z}_1 \mathcal{L}'_1 \mathfrak{H} \end{cases}$$

$$\begin{cases} {}^h\mathfrak{z}_M^J & = {}^h\mathfrak{z}_M^L {}^h\mathfrak{z}_L^J = {}^h\mathfrak{z}_M \mathcal{L}^J \\ {}^h\mathfrak{z}_M^J & = {}^h\mathfrak{z}_M^L {}^h\mathfrak{z}_L^J = {}^h\mathfrak{z}_M \mathcal{L}^J \end{cases}$$

$$\begin{cases} {}^h\mathfrak{z}_1', 1 & = \mathcal{L}'_1 {}^h\mathfrak{z}_1', 1 = {}^h\mathfrak{z}_1 \mathcal{L}'_1, 1 \\ {}^h\mathfrak{z}_1', 1 & = \mathcal{L}'_1 {}^h\mathfrak{z}_1', 1 = {}^h\mathfrak{z}_1 \mathcal{L}'_1, 1 \end{cases}$$

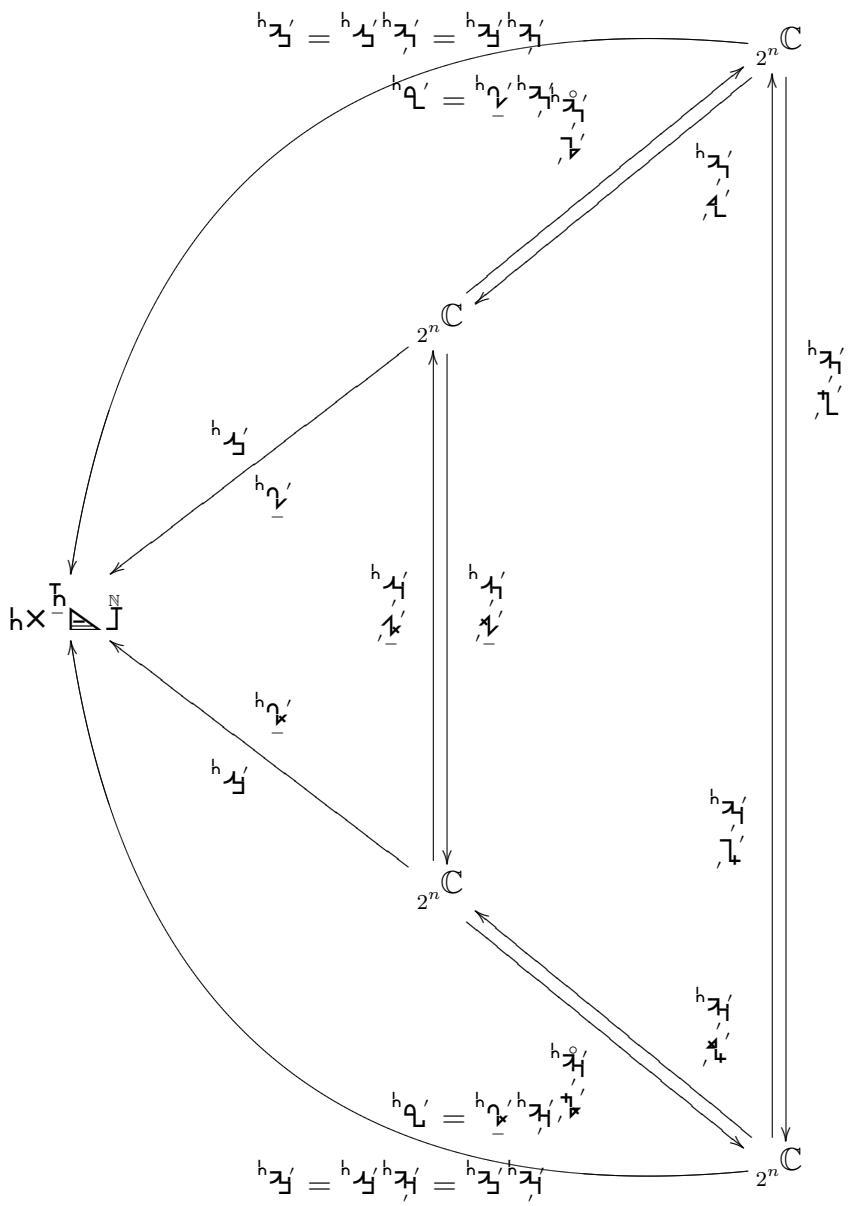
$$\begin{cases} {}^h\mathfrak{z}_I^{\circ N} & = {}^h\mathfrak{z}_I^L {}^h\mathfrak{z}_L^{\circ N} = {}^h\mathfrak{z}_I \mathcal{L}^N \\ {}^h\mathfrak{z}_I^{\circ N} & = {}^h\mathfrak{z}_I^L {}^h\mathfrak{z}_L^{\circ N} = {}^h\mathfrak{z}_I \mathcal{L}^N \end{cases}$$



$h \times \underline{\gamma}^N \ni \underline{\gamma}^J$ holonomic basis

$$, \underline{1} = \underline{\mathcal{U}}_h \underline{\gamma}^N, \underline{1}$$

$$_M \delta^N = {}_M \underline{\mathcal{U}}_h \underline{\gamma}^N$$



$$h \times \overline{h} \otimes \mathbb{I}^{\mathbb{N}} \ni \begin{cases} {}^h \mathfrak{h}^J & * \\ {}^h \mathfrak{q}^J & = \sum_j \mathfrak{h} \mathfrak{q}^j \end{cases} \text{ dual ONbasis}$$

$${}^h \mathfrak{q}^I \star {}^h \mathfrak{q}^J = {}_I \eta^J$$

$$\star {}_I \mathfrak{h}_h = {}^h \mathfrak{q}^I {}_I \eta^I$$

$${}^h \mathfrak{q}^I = (\star {}_I \mathfrak{h}_h) {}_I \eta^I$$

$$*\mathbf{^h}\mathbf{q}^I = \mathbf{^h}\mathbf{L}^{N\leftarrow I} \overline{I > N \leftarrow I}_I \eta^I$$

$$\mathbf{v} = \begin{cases} \mathbf{^h}\mathbf{\bar{x}} \mathbf{^h}\mathbf{\bar{q}'}, \mathbf{h} \\ \mathbf{^h}\mathbf{\bar{b}} \mathbf{^h}\mathbf{\bar{q}'}, \mathbf{h} \end{cases}$$

$$_I \delta^J = \begin{cases} \mathbf{^h}\mathbf{\bar{x}}^I \mathbf{^h}\mathbf{\bar{q}}^J \\ \mathbf{^h}\mathbf{\bar{b}}_h \mathbf{^h}\mathbf{\bar{q}}^J \end{cases}$$

$$\begin{cases} \mathbf{^h}\mathbf{\bar{x}}' \mathbf{h} \\ \mathbf{^h}\mathbf{\bar{q}}' \mathbf{h} \end{cases} = \begin{cases} \mathbf{^h}\mathbf{\bar{y}}' \mathbf{^h}\mathbf{\bar{x}}' \mathbf{h} \\ \mathbf{^h}\mathbf{\bar{y}}' \mathbf{^h}\mathbf{\bar{q}}' \mathbf{h} \end{cases}$$

$$\begin{cases} \mathbf{^h}\mathbf{\bar{x}}^J \\ \mathbf{^h}\mathbf{\bar{q}}^J \end{cases} = \begin{cases} \mathbf{^h}\mathbf{\bar{y}}^L \mathbf{^h}\mathbf{\bar{x}}^J \\ \mathbf{^h}\mathbf{\bar{y}}^L \mathbf{^h}\mathbf{\bar{q}}^J \end{cases}$$

$$\mathbf{^h}\mathbf{\bar{y}}', \mathbf{1} = \begin{cases} \mathbf{^h}\mathbf{\bar{x}}' \mathbf{^h}\mathbf{\bar{x}}', \mathbf{1} \\ \mathbf{^h}\mathbf{\bar{q}}', \mathbf{^h}\mathbf{\bar{q}}, \mathbf{1} \end{cases}$$

$$\mathbf{^h}\mathbf{\bar{y}}^N = \begin{cases} \mathbf{^h}\mathbf{\bar{x}}^K \mathbf{^h}\mathbf{\bar{x}}^N \\ \mathbf{^h}\mathbf{\bar{q}}^K \mathbf{^h}\mathbf{\bar{q}}^N \end{cases}$$

$$\begin{cases} \mathbf{^h}\mathbf{\bar{x}}' \mathbf{h} \\ \mathbf{^h}\mathbf{\bar{q}}' \mathbf{h} \end{cases} = \begin{cases} \mathbf{^h}\mathbf{\bar{y}}_h \mathbf{^h}\mathbf{\bar{x}}' \mathbf{h} \\ \mathbf{^h}\mathbf{\bar{y}}_h \mathbf{^h}\mathbf{\bar{q}}' \mathbf{h} \end{cases}$$

$$\begin{cases} \mathbf{^h}\mathbf{\bar{x}}^J \\ \mathbf{^h}\mathbf{\bar{q}}^J \end{cases} = \begin{cases} \mathbf{^h}\mathbf{\bar{y}}_h \mathbf{^h}\mathbf{\bar{x}}^J \\ \mathbf{^h}\mathbf{\bar{y}}_h \mathbf{^h}\mathbf{\bar{q}}^J \end{cases}$$

$$\begin{cases} \mathbf{^h}\mathbf{\bar{x}}', \mathbf{1} \\ \mathbf{^h}\mathbf{\bar{q}}', \mathbf{1} \end{cases} = \begin{cases} \mathbf{^h}\mathbf{\bar{x}}' \mathbf{^h}\mathbf{\bar{y}}', \mathbf{1} \\ \mathbf{^h}\mathbf{\bar{b}}_h \mathbf{^h}\mathbf{\bar{y}}', \mathbf{1} \end{cases}$$

$$\begin{cases} \mathbf{^h}\mathbf{\bar{x}}^N \\ \mathbf{^h}\mathbf{\bar{q}}^N \end{cases} = \begin{cases} \mathbf{^h}\mathbf{\bar{x}}^I \mathbf{^h}\mathbf{\bar{y}}^N \\ \mathbf{^h}\mathbf{\bar{b}}_h \mathbf{^h}\mathbf{\bar{y}}^N \end{cases}$$

