

$$\begin{array}{c}
\begin{array}{c|cc|c}
a & x & \xi \\
\hline
0 & -a^* & 0 \\
0 & \xi^* & 0
\end{array} \\
\hline
\begin{array}{c|cc|c}
a & x & \xi & b & y & \eta \\
\hline
0 & -a^* & 0 & 0 & -b^* & 0 \\
0 & \xi^* & 0 & 0 & \eta^* & 0
\end{array} * = \frac{\begin{array}{c|cc|c}
ab - ba & ay + ya^* - xb^* - bx + \xi\eta^* + \eta\xi^* & a\eta - b\xi \\
\hline
0 & a^*b^* - b^*a^* & 0 \\
0 & \eta^*a^* - \xi^*b^* & 0
\end{array}}{\begin{array}{c|cc|c}
0 & x & \xi & 0 & y & \eta \\
\hline
0 & 0 & 0 & 0 & 0 & 0 \\
0 & \xi^* & 0 & 0 & \eta^* & 0
\end{array}} = \frac{\begin{array}{c|cc|c}
0 & \xi\eta^* + \eta\xi^* & 0 \\
\hline
0 & 0 & 0 \\
0 & 0 & 0
\end{array}}{0}
\end{array}$$

$$\xi^* \sigma^\mu \eta = 2\overline{i\sigma^2\xi} \overline{i\sigma^2\eta}^T$$

$$\begin{aligned}
x^0 &= \xi^* \sigma^0 \eta = \begin{bmatrix} \bar{\xi}_1 & \bar{\xi}_2 \end{bmatrix} \frac{1}{0} \left| \begin{array}{c|c} 0 & 1 \\ 0 & 1 \end{array} \right| \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} = \bar{\xi}_1 \eta_1 + \bar{\xi}_2 \eta_2 \\
x^1 &= \xi^* \sigma^1 \eta = \begin{bmatrix} \bar{\xi}_1 & \bar{\xi}_2 \end{bmatrix} \frac{0}{1} \left| \begin{array}{c|c} 1 & 0 \\ 0 & 0 \end{array} \right| \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} = \bar{\xi}_2 \eta_1 + \bar{\xi}_1 \eta_2 \\
x^2 &= \xi^* \sigma^2 \eta = \begin{bmatrix} \bar{\xi}_1 & \bar{\xi}_2 \end{bmatrix} \frac{0}{i} \left| \begin{array}{c|c} -i & 0 \\ i & 0 \end{array} \right| \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} = i \bar{\xi}_2 \eta_1 - i \bar{\xi}_1 \eta_2 \\
x^3 &= \xi^* \sigma^3 \eta = \begin{bmatrix} \bar{\xi}_1 & \bar{\xi}_2 \end{bmatrix} \frac{1}{0} \left| \begin{array}{c|c} 0 & -1 \\ 0 & 1 \end{array} \right| \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} = \bar{\xi}_1 \eta_1 - \bar{\xi}_2 \eta_2 \\
\Rightarrow \text{LHS} &= \frac{x^0 - x^3}{-x^1 - ix^2} \left| \begin{array}{c|c} -x^1 + ix^2 & x^0 + x^3 \\ x^0 + x^3 & -x^1 - ix^2 \end{array} \right| = \frac{\bar{\xi}_1 \eta_1 + \bar{\xi}_2 \eta_2 - \bar{\xi}_1 \eta_1 + \bar{\xi}_2 \eta_2}{-\bar{\xi}_2 \eta_1 - \bar{\xi}_1 \eta_2 - i \underbrace{\bar{\xi}_2 \eta_1}_{\bar{\xi}_1 \eta_2} - i \underbrace{\bar{\xi}_1 \eta_2}_{\bar{\xi}_2 \eta_1}} \left| \begin{array}{c|c} -\bar{\xi}_2 \eta_1 - \bar{\xi}_1 \eta_2 + i \underbrace{i \bar{\xi}_2 \eta_1}_{i \bar{\xi}_1 \eta_2} - i \bar{\xi}_1 \eta_2 & -\bar{\xi}_2 \eta_1 - \bar{\xi}_1 \eta_2 + i \underbrace{i \bar{\xi}_2 \eta_1}_{i \bar{\xi}_1 \eta_2} - i \bar{\xi}_1 \eta_2 \\ \bar{\xi}_1 \eta_1 + \bar{\xi}_2 \eta_2 + \bar{\xi}_1 \eta_1 - \bar{\xi}_2 \eta_2 & \bar{\xi}_1 \eta_1 + \bar{\xi}_2 \eta_2 + \bar{\xi}_1 \eta_1 - \bar{\xi}_2 \eta_2 \end{array} \right| \\
&= 2 \frac{\bar{\xi}_2 \eta_2}{-\bar{\xi}_1 \eta_2} \left| \begin{array}{c|c} -\bar{\xi}_2 \eta_1 & -\bar{\xi}_2 \eta_1 \\ \bar{\xi}_1 \eta_1 & \bar{\xi}_1 \eta_1 \end{array} \right| = 2 \begin{bmatrix} \bar{\xi}_2 \\ -\bar{\xi}_1 \end{bmatrix} \begin{bmatrix} \eta_2 & -\eta_1 \end{bmatrix} = 2 \overline{i\sigma^2\xi} \overline{i\sigma^2\eta}^T
\end{aligned}$$

$$\Phi_R^* \sigma^\mu \Psi_R = 2\overline{i\sigma^2\Phi_R} \overline{i\sigma^2\Psi_R}^T = 2 \underbrace{i\sigma^2\bar{\Phi}_R}_{i\sigma^2\bar{\Psi}_R} \widehat{i\sigma^2\bar{\Psi}_R}^* = 2 \Phi_L \Psi_L^*$$

$$\begin{array}{c|cc|c}
0 & x & \Phi_L & 0 \\
0 & 0 & 0 & 0 \\
0 & \Phi_L^* & 0 & 0
\end{array} * = \frac{0}{0} \left| \begin{array}{c|cc|c}
\Phi_L \Psi_L^* + \Psi_L \Phi_L^* & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{array} \right| = \frac{0}{0} \left| \begin{array}{c|cc|c}
\Phi_R^* \sigma^\mu \Psi_R + \Psi_R^* \sigma^\mu \Phi_R & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{array} \right| = 0$$

$$\Phi_R^* \sigma^\mu \Psi_R = 2 \overline{i\sigma^2 \Phi_R} \overbrace{i\sigma^2 \Psi_R}^T = 2 \underbrace{i\sigma^2 \bar{\Phi}_R}_{\Phi_L^*} \overbrace{i\sigma^2 \bar{\Psi}_R}^* = 2 \Phi_L \Psi_L^*$$

$$\begin{array}{c|c|c|c|c|c|c} 0 & x & \Phi_L & 0 & y & \Psi_L \\ \hline 0 & 0 & 0 & * & 0 & 0 & 0 \\ \hline 0 & \Phi_L^* & 0 & 0 & \Psi_L^* & 0 & 0 \end{array} = \begin{array}{c|c|c|c|c|c|c} 0 & \Phi_L \Psi_L^* + \Psi_L \Phi_L^* & 0 \\ \hline 0 & 0 & 0 \\ \hline 0 & 0 & 0 \end{array} = \begin{array}{c|c|c|c|c|c|c} 0 & \Phi_R^* \sigma^\mu \Psi_R + \Psi_R^* \sigma^\mu \Phi_R & 0 \\ \hline 0 & 0 & 0 \\ \hline 0 & 0 & 0 \end{array}$$

SUSY generators

$$\text{Majorana } Q = Q_a = \begin{bmatrix} Q_L = Q_A \\ Q_R = \bar{Q}^A \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} \\ \begin{bmatrix} \bar{Q}^1 \\ \bar{Q}^2 \end{bmatrix} \end{bmatrix}$$

$$-i\sigma^2 = \begin{array}{c|c} 0 & -1 \\ \hline 1 & 0 \end{array}$$

$$Q_L = i\sigma^2 \bar{Q}_R$$

$$Q_R = -i\sigma^2 \bar{Q}_L$$

$$\bar{Q}_{\dot{A}} = Q_{\dot{A}}^*$$

$$\bar{Q}^{\dot{A}} = (-i\sigma^2)^{\dot{A}\dot{B}} \bar{Q}_{\dot{B}} = (-i\sigma^2)^{\dot{A}\dot{B}} Q_B^*$$

$$\begin{bmatrix} \bar{Q}^1 \\ \bar{Q}^2 \end{bmatrix} = \begin{array}{c|c} 0 & -1 \\ \hline 1 & 0 \end{array} \begin{bmatrix} Q^* \\ Q_2^* \\ Q_1^* \end{bmatrix} = \begin{bmatrix} -Q_2^* \\ Q_1^* \end{bmatrix}$$

$$\begin{bmatrix} Q_L \\ Q_R \end{bmatrix} = \begin{bmatrix} i\sigma^2 \bar{Q}_R \\ -i\sigma^2 \bar{Q}_L \end{bmatrix} = \begin{array}{c|c} i\sigma^2 & 0 \\ \hline 0 & -i\sigma^2 \end{array} \begin{bmatrix} \bar{Q}_R \\ \bar{Q}_L \end{bmatrix}$$

$$\begin{array}{c|c|c|c|c|c|c} 0 & x & \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} & 0 & y & \begin{bmatrix} \tilde{Q}_1 \\ \tilde{Q}_2 \end{bmatrix} & 0 \\ \hline 0 & 0 & 0 & * & 0 & 0 & 0 \\ \hline 0 & \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix}^* & 0 & 0 & \begin{bmatrix} \tilde{Q}_1 \\ \tilde{Q}_2 \end{bmatrix} L^* & 0 & 0 \end{array} = \begin{array}{c|c|c|c|c|c|c} 0 & \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} \begin{bmatrix} \tilde{Q}_1 \\ \tilde{Q}_2 \end{bmatrix}^* & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

$$= \frac{0 \left| \begin{bmatrix} -Q_2^* \\ Q_1^* \end{bmatrix}^* \sigma^\mu \begin{bmatrix} -\tilde{Q}_2^* \\ \tilde{Q}_1^* \end{bmatrix} + \begin{bmatrix} -\tilde{Q}_2^* \\ \tilde{Q}_1^* \end{bmatrix}^* \sigma^\mu \begin{bmatrix} -Q_2^* \\ Q_1^* \end{bmatrix} \right| 0}{\begin{array}{c|c|c} 0 & 0 & 0 \\ \hline 0 & 0 & 0 \\ \hline 0 & 0 & 0 \end{array}}$$

$$= \frac{0 \left| \begin{bmatrix} -Q_2 & Q_1 \end{bmatrix} \sigma^\mu \begin{bmatrix} -\tilde{Q}_2^* \\ \tilde{Q}_1^* \end{bmatrix} + \begin{bmatrix} -\tilde{Q}_2 & \tilde{Q}_1 \end{bmatrix} \sigma^\mu \begin{bmatrix} -Q_2^* \\ Q_1^* \end{bmatrix} \right| 0}{\begin{array}{c|c} 0 & 0 \\ \hline 0 & 0 \end{array}} = \frac{0}{0}$$