

$$\begin{aligned}
& {}^x \boxed{\mathfrak{b}|\mathfrak{N}}_a^i = \frac{d}{dt} {}^x \boxed{\mathfrak{N}_t|\mathfrak{N}}_a^i \\
& {}^x \boxed{\mathfrak{b}|\mathfrak{N}:\mathfrak{N}}_{\mu}^i = \frac{d}{dt} {}^x \boxed{\mathfrak{N}_t|\mathfrak{N}:\mathfrak{N}}_{\mu}^i = \frac{d}{dt} {}^x \mathfrak{N}_t^{\nu} \overbrace{{}^x \boxed{\mathfrak{N}_t|\mathfrak{N}}_{\nu}^i + {}^x \boxed{\mathfrak{N}_t|\mathfrak{N}}_j^i}^{b}{}_{\nu} \mathfrak{N}_b^j \\
& = {}^x \mathfrak{b}_{\mu}^{\nu} {}_{\nu} \mathfrak{N}_a^i + {}_{\mu} \delta^{\nu} \overbrace{{}^x \boxed{\mathfrak{b}|\mathfrak{N}}_{\nu}^i + {}^x \boxed{\mathfrak{b}|\mathfrak{N}}_j^i}^{b} {}_{\nu} \mathfrak{N}_b^j = {}^x \mathfrak{b}_{\mu}^{\nu} {}_{\nu} \mathfrak{N}_a^i + {}^x \boxed{\mathfrak{b}|\mathfrak{N}}_{\mu}^i + {}^x \boxed{\mathfrak{b}|\mathfrak{N}}_j^i {}_{\mu} \mathfrak{N}_b^j
\end{aligned}$$