$$\Upsilon = \overset{\circ}{\Upsilon} + \vartheta_- \overset{-}{\Upsilon} + \vartheta_+ \overset{+}{\Upsilon} + \vartheta_+ \vartheta_- \overset{\bullet}{\Upsilon}$$

$${}^{x:\vartheta}\Upsilon = {}^{x\varnothing}\Upsilon + \vartheta_- {}^{x-}\Upsilon + \vartheta_+ {}^{x+}\Upsilon + \vartheta_+ \vartheta_- {}^{x\pm}\Upsilon$$

$$\text{even coord } x^\alpha : {}_\alpha\Upsilon$$

$$\text{odd coord } \vartheta_\pm : {}_\pm\Upsilon$$

$${}^{x:\vartheta}\boxed{y{:}\,{}_{\alpha:\pm}\Upsilon} = i\,{}_+\Upsilon\,{}_-\Upsilon + {}_+\Upsilon\,\vartheta_-\underbrace{{}_0\Upsilon + {}_1\Upsilon}_{} + \vartheta_+\,{}_-\Upsilon\underbrace{{}_0\Upsilon - {}_1\Upsilon}_{} - i\,\vartheta_+\vartheta_-\underbrace{{}_0\Upsilon^2 - {}_1\Upsilon^2}_{}$$

$$D = \begin{bmatrix} -i\partial_+ - \vartheta_+\underbrace{\partial_0 - \partial_1}_{} \\ i\partial_- + \vartheta_-\underbrace{\partial_0 + \partial_1}_{} \end{bmatrix}$$

$$\bar{D} = \begin{bmatrix} \partial_- - i\vartheta_-\underbrace{\partial_0 + \partial_1}_{} & \partial_+ - i\vartheta_+\underbrace{\partial_0 - \partial_1}_{} \end{bmatrix}$$

$$\text{LHS} = \frac{1}{2}\bar{D}\Upsilon D\Upsilon = \frac{1}{2}\begin{bmatrix} {}_-\Upsilon - i\vartheta_-\underbrace{{}_0\Upsilon + {}_1\Upsilon}_{} & {}_+\Upsilon - i\vartheta_+\underbrace{{}_0\Upsilon - {}_1\Upsilon}_{} \end{bmatrix}\begin{bmatrix} -i\,{}_+\Upsilon - \vartheta_+\underbrace{{}_0\Upsilon - {}_1\Upsilon}_{} \\ i\,{}_-\Upsilon + \vartheta_-\underbrace{{}_0\Upsilon + {}_1\Upsilon}_{} \end{bmatrix}$$

$$= \frac{1}{2}\left({}_-\Upsilon - i\vartheta_-\underbrace{{}_0\Upsilon + {}_1\Upsilon}_{}\right)\left(-i\,{}_+\Upsilon - \vartheta_+\underbrace{{}_0\Upsilon - {}_1\Upsilon}_{}\right) + \left({}_+\Upsilon - i\vartheta_+\underbrace{{}_0\Upsilon - {}_1\Upsilon}_{}\right)\left(i\,{}_-\Upsilon + \vartheta_-\underbrace{{}_0\Upsilon + {}_1\Upsilon}_{}\right) = \text{RHS}$$

$$i\,\vartheta_-\vartheta_+\left(\partial_0^2 - \partial_1^2\right)Y + \vartheta_+\partial_-\left(\partial_0 - \partial_1\right)Y - \vartheta_-\partial_+\left(\partial_0 + \partial_1\right)Y - i\,\partial_-\partial_+ Y \underset{\text{motion}}{=} 0$$

$$\begin{cases} {}^{x:\vartheta}\boxed{y{:}\,{}_{\alpha:\pm}\Upsilon}_- = -i\,{}_+\Upsilon - \vartheta_+\underbrace{{}_0\Upsilon - {}_1\Upsilon}_{} & {}^{x:\vartheta}\boxed{y{:}\,{}_{\alpha:\pm}\Upsilon}_+ = i\,{}_-\Upsilon + \vartheta_-\underbrace{{}_0\Upsilon + {}_1\Upsilon}_{} \\ {}^{x:\vartheta}\boxed{y{:}\,{}_{\alpha:\pm}\Upsilon}_0 = {}_+\Upsilon\vartheta_- + \vartheta_+\,{}_-\Upsilon - 2i\vartheta_+\vartheta_-\,{}_0\Upsilon & {}^{x:\vartheta}\boxed{y{:}\,{}_{\alpha:\pm}\Upsilon}_1 = {}_+\Upsilon\vartheta_- - \vartheta_+\,{}_-\Upsilon + 2i\vartheta_+\vartheta_-\,{}_1\Upsilon \end{cases}$$

$$\Rightarrow 0 = {}_-\overline{-i\,{}_{+}\underline{\Upsilon} - \vartheta_+\underbrace{{}_{0}\underline{\Upsilon} - {}_{1}\underline{\Upsilon}}_{}} + {}_+\overline{i\,{}_{-}\underline{\Upsilon} + \vartheta_-\underbrace{{}_{0}\underline{\Upsilon} + {}_{1}\underline{\Upsilon}}_{}} + {}_0\overline{{}_{+}\underline{\Upsilon}\vartheta_- + \vartheta_+\,{}_{-}\underline{\Upsilon} - 2i\vartheta_+\vartheta_-\,{}_{0}\underline{\Upsilon}} + {}_1\overline{{}_{+}\underline{\Upsilon}\vartheta_- - \vartheta_+\,{}_{-}\underline{\Upsilon} + 2i\vartheta_+\vartheta_-\,{}_{1}\underline{\Upsilon}}$$

$$= -i\,{}_{-+}\underline{\underline{\Upsilon}} + \vartheta_+\underbrace{{}_{-0}\underline{\underline{\Upsilon}} - {}_{-1}\underline{\underline{\Upsilon}}}_{} + i\,{}_{+-}\underline{\underline{\Upsilon}} - \vartheta_-\underbrace{{}_{+0}\underline{\underline{\Upsilon}} + {}_{+1}\underline{\underline{\Upsilon}}}_{} + {}_{0+}\underline{\underline{\Upsilon}}\,\vartheta_- + \vartheta_+\,{}_{0-}\underline{\underline{\Upsilon}}$$

$$-2i\,\vartheta_+\vartheta_-\,{}_{00}\underline{\underline{\Upsilon}} + {}_{1+}\underline{\underline{\Upsilon}}\,\vartheta_- - \vartheta_+\,{}_{1-}\underline{\underline{\Upsilon}} + 2i\,\vartheta_+\vartheta_-\,{}_{11}\underline{\underline{\Upsilon}}$$

$$= -2i\,\vartheta_+\vartheta_-\overbrace{{}_{00}\underline{\underline{\Upsilon}} - {}_{11}\underline{\underline{\Upsilon}}}^{} + 2i\,{}_{+-}\underline{\underline{\Upsilon}} + 2\,\vartheta_+\underbrace{{}_{-0}\underline{\underline{\Upsilon}} - {}_{-1}\underline{\underline{\Upsilon}}}_{} - 2\,\vartheta_-\underbrace{{}_{+0}\underline{\underline{\Upsilon}} + {}_{+1}\underline{\underline{\Upsilon}}}_{}$$

$$\partial^{\alpha}\,\partial_{\alpha}\,\overset{\circ}{\mathsf{H}} = 0 = \overset{\bullet}{\mathsf{H}}$$

$$\varrho^{\alpha}\,\partial_{\alpha}\,\overset{\pm}{\mathsf{H}} = 0$$

$$\overset{\bullet}{\mathsf{H}} = 0$$

$$\left(\partial_0^2 - \partial_1^2\right)\overset{\circ}{\mathsf{H}} = 0 = \partial^{\alpha}\,\partial_{\alpha}\,\overset{\circ}{\mathsf{H}}$$

$$\left(\partial_0 + \partial_1\right)\overset{+}{\mathsf{H}} = 0 = \left(\partial_0 - \partial_1\right)\overset{-}{\mathsf{H}} \Leftrightarrow \varrho^{\alpha}\,\partial_{\alpha}\,\overset{\pm}{\mathsf{H}} = 0$$

$$\overset{x:\vartheta}{}\boxed{y:{}_{\alpha:\pm}\mathsf{Y}} = \underbrace{{}_{-}\mathsf{Y} - i\vartheta_{-}\,{}_{\alpha}\mathsf{Y}}\,\underbrace{-i\,{}_{+}\mathsf{Y} - (-1)^{\beta}\vartheta_{+}\,{}_{\beta}\mathsf{Y}} + \underbrace{{}_{+}\mathsf{Y} - i(-1)^{\alpha}\vartheta_{+}\,{}_{\alpha}\mathsf{Y}}\,\underbrace{i\,{}_{-}\mathsf{Y} + \vartheta_{-}\,{}_{\beta}\mathsf{Y}}$$

$$= 2i\,{}_{+}\mathsf{Y}\,{}_{-}\mathsf{Y} + 2\,{}_{+}\mathsf{Y}\,\vartheta_{-}\,{}_{\alpha}\mathsf{Y} + 2\,(-1)^{\alpha}\,\vartheta_{+}\,{}_{-}\mathsf{Y}\,{}_{\alpha}\mathsf{Y} - i\left(1-(-1)^{\alpha}\right)\vartheta_{+}\,\vartheta_{-}\,{}_{\alpha}\mathsf{Y}\,{}_{\beta}\mathsf{Y}$$

$$\psi = \begin{matrix}\psi_{-}\\ \psi_{+}\end{matrix}$$

$$\bar{\psi} = \overset{*}{\psi}\,\varrho^{0} = \overset{*}{\psi}\begin{array}{c|c}0 & -i\\ \hline i & 0\end{array} = \begin{bmatrix}\psi_{+}i & -\psi_{-}i\end{bmatrix}$$

$$\bar{\vartheta}\,\vartheta = \begin{bmatrix}\vartheta_{+}i & -\vartheta_{-}i\end{bmatrix}\begin{matrix}\vartheta_{-}\\ \vartheta_{+}\end{matrix} = 2\,\vartheta_{+}\,\vartheta_{-}\,i$$

$$\bar{\vartheta} = \begin{bmatrix}\vartheta_{+}i & -\vartheta_{-}i\end{bmatrix}$$

$$D = \frac{\partial}{\partial\bar{\vartheta}} - i\,\varrho^{\alpha}\,\vartheta\,\partial_{\alpha} = \begin{bmatrix}\frac{1}{i}\partial_{+}\\ -\frac{1}{i}\partial_{-}\end{bmatrix} - i\begin{array}{c|c}0 & -(-1)^{\alpha}i\\ \hline i & 0\end{array}\begin{bmatrix}\vartheta_{-}\\ \vartheta_{+}\end{bmatrix}\partial_{\alpha} = \begin{bmatrix}-i\partial_{+} - (-1)^{\alpha}\vartheta_{+}\partial_{\alpha}\\ i\partial_{-} + \vartheta_{-}\partial_{\alpha}\end{bmatrix}$$

$$\bar{D} = \frac{\partial}{\partial\vartheta} - i\,\bar{\vartheta}\,\varrho^{\alpha}\,\partial_{\alpha} = \begin{bmatrix}\partial_{-} & \partial_{+}\end{bmatrix} - i\begin{bmatrix}\vartheta_{+}i & -\vartheta_{-}i\end{bmatrix}\begin{array}{c|c}0 & -(-1)^{\alpha}i\\ \hline i & 0\end{array}\partial_{\alpha}$$

$$= \begin{bmatrix}\partial_{-} & \partial_{+}\end{bmatrix} - i\begin{bmatrix}\vartheta_{-} & (-1)^{\alpha}\vartheta_{+}\end{bmatrix}\partial_{\alpha} = \begin{bmatrix}\partial_{-} - i\vartheta_{-}\partial_{\alpha} & \partial_{+} - i(-1)^{\alpha}\vartheta_{+}\partial_{\alpha}\end{bmatrix}$$

$$\bar{D}Y DY = \begin{bmatrix}\partial_{-}Y - i\vartheta_{-}\partial_{\alpha}Y & \partial_{+}Y - i(-1)^{\alpha}\vartheta_{+}\partial_{\alpha}Y\end{bmatrix}\begin{bmatrix}-i\partial_{+}Y - (-1)^{\beta}\vartheta_{+}\partial_{\beta}Y\\ i\partial_{-}Y + \vartheta_{-}\partial_{\beta}Y\end{bmatrix}$$

$$= \underbrace{\partial_{-}Y - i\vartheta_{-}\partial_{\alpha}Y}\,\underbrace{-i\partial_{+}Y - (-1)^{\beta}\vartheta_{+}\partial_{\beta}Y} + \underbrace{\partial_{+}Y - i(-1)^{\alpha}\vartheta_{+}\partial_{\alpha}Y}\,\underbrace{i\partial_{-}Y + \vartheta_{-}\partial_{\beta}Y}$$

$$\bar{D}Y = \bar{\psi} + B\,\bar{\vartheta} + i\,{}_{\alpha}X\,\bar{\vartheta}\,\varrho^{\alpha} - \frac{i}{2}\,\bar{\vartheta}\,\vartheta\,{}_{\alpha}\bar{\psi}\,\varrho^{\alpha}$$

$$= \begin{bmatrix}\psi_{+}i & -\psi_{-}i\end{bmatrix} + B\begin{bmatrix}\vartheta_{+}i & -\vartheta_{-}i\end{bmatrix} + i\,{}_{\alpha}X\begin{bmatrix}\vartheta_{+}i & -\vartheta_{-}i\end{bmatrix}\begin{array}{c|c}0 & -(-1)^{\alpha}i\\ \hline i & 0\end{array} + \vartheta_{+}\,\vartheta_{-}\begin{bmatrix}{}_{\alpha}\psi_{+}i & -{}_{\alpha}\psi_{-}i\end{bmatrix}\begin{array}{c|c}0 & -(-1)^{\alpha}i\\ \hline i & 0\end{array}$$

$$= \begin{bmatrix}\psi_{+}i & -\psi_{-}i\end{bmatrix} + B\begin{bmatrix}\vartheta_{+}i & -\vartheta_{-}i\end{bmatrix} + i\,{}_{\alpha}X\begin{bmatrix}\vartheta_{-} & (-1)^{\alpha}\vartheta_{+}\end{bmatrix} + \vartheta_{+}\,\vartheta_{-}\begin{bmatrix}{}_{\alpha}\psi_{-} & (-1)^{\alpha}\,{}_{\alpha}\psi_{+}\end{bmatrix}$$

$$= \begin{bmatrix}\psi_{+}i + B\vartheta_{+}i + i_{\alpha}X\vartheta_{-} + \vartheta_{+}\vartheta_{-\alpha}\psi_{-} & -\psi_{-}i - B\vartheta_{-}i + i_{\alpha}X(-1)^{\alpha}\vartheta_{+} + \vartheta_{+}\vartheta_{-}(-1)^{\alpha}\,{}_{\alpha}\psi_{+}\end{bmatrix}$$

$$DY = \psi + \vartheta B - i\,\varrho^{\beta}\,\vartheta\,{}_{\beta}X + \frac{i}{2}\,\bar{\vartheta}\,\vartheta\,\varrho^{\beta}\,{}_{\beta}\psi$$

$$= \begin{bmatrix} \psi_{-} \\ \psi_{+} \end{bmatrix} + \begin{bmatrix} \vartheta_{-} \\ \vartheta_{+} \end{bmatrix} B - i \begin{array}{c|c} 0 & -(-1)^{\beta} i \\ \hline i & 0 \end{array} \begin{bmatrix} \vartheta_{-} \\ \vartheta_{+} \end{bmatrix} {}_{\beta}X - \vartheta_{+} \vartheta_{-} \begin{array}{c|c} 0 & -(-1)^{\beta} i \\ \hline i & 0 \end{array} \begin{bmatrix} {}_{\beta}\psi_{-} \\ {}_{\beta}\psi_{+} \end{bmatrix}$$

$$= \begin{bmatrix} \psi_{-} \\ \psi_{+} \end{bmatrix} + \begin{bmatrix} \vartheta_{-} \\ \vartheta_{+} \end{bmatrix} B - i \begin{bmatrix} -(-1)^{\beta} \vartheta_{+} i \\ \vartheta_{-} i \end{bmatrix} {}_{\beta}X - \vartheta_{+} \vartheta_{-} \begin{bmatrix} -(-1)^{\beta} {}_{\beta}\psi_{+} i \\ {}_{\beta}\psi_{-} i \end{bmatrix}$$

$$= \begin{bmatrix} \psi_{-} + \vartheta_{-} B - (-1)^{\beta} \vartheta_{+\beta} X + \vartheta_{+} \vartheta_{-} (-1)^{\beta} {}_{\beta}\psi_{+} i \\ \psi_{+} + \vartheta_{+} B + \vartheta_{-\beta} X - \vartheta_{+} \vartheta_{-\beta} \psi_{-} i \end{bmatrix}$$

$$\bar{D} Y D Y = \underbrace{\psi_{+} i + B \vartheta_{+} i + i_{\alpha} X \vartheta_{-} + \vartheta_{+} \vartheta_{-\alpha} \psi_{-}} \underbrace{\psi_{-} + \vartheta_{-} B - (-1)^{\beta} \vartheta_{+\beta} X + \vartheta_{+} \vartheta_{-} (-1)^{\beta} {}_{\beta}\psi_{+} i}$$

$$+ \underbrace{-\psi_{-} i - B \vartheta_{-} i + i_{\alpha} X (-1)^{\alpha} \vartheta_{+} + \vartheta_{+} \vartheta_{-} (-1)^{\alpha} {}_{\alpha}\psi_{+}} \underbrace{\psi_{+} + \vartheta_{+} B + \vartheta_{-\beta} X - \vartheta_{+} \vartheta_{-\beta} \psi_{-} i}$$

$$= \psi_{+} i \underbrace{\psi_{-} + \vartheta_{-} B - (-1)^{\beta} \vartheta_{+\beta} X + \vartheta_{+} \vartheta_{-} (-1)^{\beta} {}_{\beta}\psi_{+} i} + B \vartheta_{+} i \underbrace{\psi_{-} + \vartheta_{-} B} + i_{\alpha} X \vartheta_{-} \underbrace{\psi_{-} - (-1)^{\beta} \vartheta_{+\beta} X} + \vartheta_{+} \vartheta_{-\alpha} \psi_{-} \psi_{-}$$

$$- \psi_{-} i \underbrace{\psi_{+} + \vartheta_{+} B + \vartheta_{-\beta} X - \vartheta_{+} \vartheta_{-\beta} \psi_{-} i} - B \vartheta_{-} i \underbrace{\psi_{+} + \vartheta_{+} B} + i_{\alpha} X (-1)^{\alpha} \vartheta_{+} \underbrace{\psi_{+} + \vartheta_{-\beta} X} + \vartheta_{+} \vartheta_{-} (-1)^{\alpha} {}_{\alpha}\psi_{+} \psi_{+}$$