

$$\begin{aligned}\partial_A &= \frac{\partial}{\partial \zeta^A} \\ \partial_A \star \partial_B &= 0 \\ \partial_A \star \bar{\zeta}^{\dot{D}} &= 0 \\ \partial_A \star \zeta^B &= \delta^B_A\end{aligned}$$

$$\begin{aligned}\partial_A \star \underbrace{\bar{\zeta}^{\dot{D}}}_{\partial_\nu} &= 0 = \underbrace{\bar{\zeta}^{\dot{B}}}_{\partial_\mu} \star \underbrace{\bar{\zeta}^{\dot{D}}}_{\partial_\nu} \\ \underbrace{\bar{\zeta}^{\dot{B}}}_{\partial_\mu} \star \zeta^C &= 0 = \underbrace{\bar{\zeta}^{\dot{B}}}_{\partial_\mu} \star \bar{\zeta}^{\dot{C}}\end{aligned}$$

$$\partial_A \star \underbrace{\bar{\zeta}^{\dot{D}}}_{\partial_\nu} = \partial_A \bar{\zeta}^{\dot{D}} \partial_\nu + \bar{\zeta}^{\dot{D}} \partial_\nu \partial_A = \partial_A \bar{\zeta}^{\dot{D}} \partial_\nu + \bar{\zeta}^{\dot{D}} \partial_A \partial_\nu = \underbrace{\partial_A \bar{\zeta}^{\dot{D}} + \bar{\zeta}^{\dot{D}} \partial_A}_{=0} \partial_\nu = \underbrace{\partial_A \star \bar{\zeta}^{\dot{D}}}_{=0} \partial_\nu = 0$$

$$\underbrace{\bar{\zeta}^{\dot{B}}}_{\partial_\mu} \star \underbrace{\bar{\zeta}^{\dot{D}}}_{\partial_\nu} = \underbrace{\bar{\zeta}^{\dot{B}}}_{\partial_\mu} \partial_\nu \bar{\zeta}^{\dot{D}} + \bar{\zeta}^{\dot{D}} \partial_\nu \bar{\zeta}^{\dot{B}} \partial_\mu = \bar{\zeta}^{\dot{B}} \bar{\zeta}^{\dot{D}} \partial_\mu \partial_\nu + \bar{\zeta}^{\dot{D}} \bar{\zeta}^{\dot{B}} \partial_\nu \partial_\mu = \underbrace{\bar{\zeta}^{\dot{B}} \bar{\zeta}^{\dot{D}} + \bar{\zeta}^{\dot{D}} \bar{\zeta}^{\dot{B}}}_{=0} \partial_\nu \partial_\mu = \underbrace{\bar{\zeta}^{\dot{B}} \star \bar{\zeta}^{\dot{D}}}_{=0} \partial_\mu \partial_\nu =$$

$$\underbrace{\bar{\zeta}^{\dot{B}}}_{\partial_\mu} \star \zeta^C = \bar{\zeta}^{\dot{B}} \partial_\mu \zeta^C + \zeta^C \bar{\zeta}^{\dot{B}} \partial_\mu = \underbrace{\bar{\zeta}^{\dot{B}} \zeta^C + \zeta^C \bar{\zeta}^{\dot{B}}}_{=0} \partial_\mu = 0$$

$$\underbrace{\bar{\zeta}^{\dot{B}}}_{\partial_\mu} \star \bar{\zeta}^{\dot{C}} = \bar{\zeta}^{\dot{B}} \partial_\mu \bar{\zeta}^{\dot{C}} + \bar{\zeta}^{\dot{C}} \bar{\zeta}^{\dot{B}} \partial_\mu = \underbrace{\bar{\zeta}^{\dot{B}} \bar{\zeta}^{\dot{C}} + \bar{\zeta}^{\dot{C}} \bar{\zeta}^{\dot{B}}}_{=0} \partial_\mu = 0$$

$$\mathbb{L}^\mu = i \sigma^\mu$$

$$Q_A = \partial_A - \underbrace{\mathbb{L}^\mu \bar{\zeta}}_A \partial_\mu = \partial_A - \mathbb{L}^\mu_{AB} \bar{\zeta}^{\dot{B}} \partial_\mu$$

$$D_A = \partial_A + \underbrace{\mathbb{L}^\mu \bar{\zeta}}_A \partial_\mu = \partial_A + \mathbb{L}^\mu_{AB} \bar{\zeta}^{\dot{B}} \partial_\mu$$

$$Q_A \star Q_C = D_A \star D_C = Q_A \star D_C = 0$$

$$Q_A \star \zeta^C = {}_A \delta^C = D_A \star \zeta^C$$

$$Q_A \star \bar{\zeta}^{\dot{C}} = 0 = D_A \star \bar{\zeta}^{\dot{C}}$$

$$\begin{aligned} & \underbrace{\partial_A + \alpha \mathbb{L}_{AB}^\mu \bar{\zeta}^{\dot{B}} \partial_\mu}_{=} \star \underbrace{\partial_C + \beta \mathbb{L}_{CD}^\nu \bar{\zeta}^{\dot{D}} \partial_\nu}_{=} \\ = & \underbrace{\partial_A \star \partial_C}_{=0} + \beta \underbrace{\partial_A \star \bar{\zeta}^{\dot{D}} \partial_\nu}_{=0} \mathbb{L}_{CD}^\nu + \alpha \mathbb{L}_{AB}^\mu \underbrace{\bar{\zeta}^{\dot{B}} \partial_\mu \star \partial_C}_{=0} + \alpha \beta \mathbb{L}_{AB}^\mu \underbrace{\bar{\zeta}^{\dot{B}} \partial_\mu \star \bar{\zeta}^{\dot{D}} \partial_\nu}_{=0} \mathbb{L}_{CD}^\nu = 0 \\ \underbrace{\partial_A + \alpha \mathbb{L}_{AB}^\mu \bar{\zeta}^{\dot{B}} \partial_\mu}_{=} \star \zeta^C & = \underbrace{\partial_A \star \zeta^C}_{= {}_A \delta^C} + \alpha \mathbb{L}_{AB}^\mu \underbrace{\bar{\zeta}^{\dot{B}} \partial_\mu \star \zeta^C}_{=0} = {}_A \delta^C \\ \underbrace{\partial_A + \alpha \mathbb{L}_{AB}^\mu \bar{\zeta}^{\dot{B}} \partial_\mu}_{=} \star \bar{\zeta}^{\dot{C}} & = \underbrace{\partial_A \star \bar{\zeta}^{\dot{C}}}_{=0} + \alpha \mathbb{L}_{AB}^\mu \underbrace{\bar{\zeta}^{\dot{B}} \partial_\mu \star \bar{\zeta}^{\dot{C}}}_{=0} = 0 \end{aligned}$$

$$\bar{\partial}_{\dot{A}} = \frac{\partial}{\partial \bar{\zeta}^{\dot{A}}}$$

$$\bar{\partial}_{\dot{A}} \star \bar{\partial}_{\dot{B}} = 0$$

$$\bar{\partial}_{\dot{A}} \star \bar{\zeta}^{\dot{B}} = {}_{\dot{A}} \delta^{\dot{B}}$$

$$\bar{\partial}_{\dot{A}} \star \zeta^D \partial_\nu = 0 = \zeta^B \partial_\mu \star \zeta^D \partial_\nu$$

$$\zeta^B \partial_\mu \star \bar{\zeta}^{\dot{C}} = 0 = \bar{\zeta}^{\dot{B}} \partial_\mu \star \zeta^C$$

$$\bar{\partial}_{\dot{A}} \star \zeta^D \partial_\nu = \bar{\partial}_{\dot{A}} \zeta^D \partial_\nu + \zeta^D \partial_\nu \bar{\partial}_{\dot{A}} = \bar{\partial}_{\dot{A}} \zeta^D \partial_\nu + \zeta^D \bar{\partial}_{\dot{A}} \partial_\nu = \underbrace{\bar{\partial}_{\dot{A}} \zeta^D + \zeta^D \bar{\partial}_{\dot{A}}}_{=0} \partial_\nu = \underbrace{\bar{\partial}_{\dot{A}} \star \zeta^D}_{=0} \partial_\nu = 0$$

$$\zeta^B \partial_\mu \star \zeta^D \partial_\nu = \zeta^B \partial_\mu \zeta^D \partial_\nu + \zeta^D \partial_\nu \zeta^B \partial_\mu = \zeta^B \zeta^D \partial_\mu \partial_\nu + \zeta^D \zeta^B \partial_\nu \partial_\mu = \underbrace{\zeta^B \zeta^D + \zeta^D \zeta^B}_{=0} \partial_\nu \partial_\mu = \underbrace{\zeta^B \star \zeta^D}_{=0} \partial_\mu \partial_\nu = 0$$

$$\zeta^B \partial_\mu \star \bar{\zeta}^{\dot{C}} = \zeta^B \partial_\mu \bar{\zeta}^{\dot{C}} + \bar{\zeta}^{\dot{C}} \zeta^B \partial_\mu = \underbrace{\zeta^B \bar{\zeta}^{\dot{C}} + \bar{\zeta}^{\dot{C}} \zeta^B}_{=0} \partial_\mu = 0$$

$$\bar{\zeta}^{\dot{B}} \partial_\mu \star \zeta^C = \bar{\zeta}^{\dot{B}} \partial_\mu \zeta^C + \zeta^C \bar{\zeta}^{\dot{B}} \partial_\mu = \underbrace{\bar{\zeta}^{\dot{B}} \zeta^C + \zeta^C \bar{\zeta}^{\dot{B}}}_{=0} \partial_\mu = 0$$

$$\bar{Q}_{\dot{A}} = \bar{\partial}_{\dot{A}} - \zeta^{\mathbb{L}^\mu} \partial_\mu = \bar{\partial}_{\dot{A}} - \zeta^B \mathbb{T}_{BA}^\mu \partial_\mu$$

$$\bar{D}_{\dot{A}} = \bar{\partial}_{\dot{A}} + \zeta^{\mathbb{L}^\mu} \partial_\mu = \bar{\partial}_{\dot{A}} + \zeta^B \mathbb{T}_{BA}^\mu \partial_\mu$$

$$\bar{Q}_{\dot{A}} \star \bar{Q}_{\dot{C}} = \bar{D}_{\dot{A}} \star \bar{D}_{\dot{C}} = \bar{Q}_{\dot{A}} \star \bar{D}_{\dot{C}} = 0$$

$$\bar{Q}_{\dot{A}} \star \zeta^C = 0 = \bar{D}_{\dot{A}} \star \zeta^C$$

$$\bar{Q}_{\dot{A}} \star \bar{\zeta}^{\dot{C}} = \delta^{\dot{C}}_{\dot{A}} = \bar{D}_{\dot{A}} \star \bar{\zeta}^{\dot{C}}$$

$$\begin{aligned} & \underbrace{\bar{\partial}_{\dot{A}} + \alpha \zeta^B \mathbb{L}_{BA}^\mu \partial_\mu}_{\text{=0}} \star \underbrace{\bar{\partial}_{\dot{C}} + \beta \zeta^D \mathbb{L}_{DC}^\nu \partial_\nu}_{\text{=0}} \\ &= \overbrace{\bar{\partial}_{\dot{A}} \star \bar{\partial}_{\dot{C}}}^{\text{=0}} + \beta \overbrace{\bar{\partial}_{\dot{A}} \star \zeta^D \partial_\nu}_{\text{=0}} \mathbb{L}_{DC}^\nu + \alpha \mathbb{L}_{BA}^\mu \overbrace{\zeta^B \partial_\mu \star \bar{\partial}_{\dot{C}}}^{\text{=0}} \\ & \quad + \alpha \beta \mathbb{L}_{BA}^\mu \overbrace{\zeta^B \partial_\mu \star \zeta^D \partial_\nu}_{\text{=0}} \mathbb{L}_{DC}^\nu = 0 \\ & \underbrace{\bar{\partial}_{\dot{A}} + \alpha \zeta^B \mathbb{L}_{BA}^\mu \partial_\mu}_{\text{=0}} \star \bar{\zeta}^{\dot{C}} + \alpha \mathbb{L}_{BA}^\mu \overbrace{\zeta^B \partial_\mu \star \zeta^{\dot{C}}}_{\text{=0}} = 0 \\ & \underbrace{\bar{\partial}_{\dot{A}} + \alpha \zeta^B \mathbb{L}_{BA}^\mu \partial_\mu}_{\text{=0}} \star \bar{\zeta}^{\dot{C}} = \overbrace{\bar{\partial}_{\dot{A}} \star \bar{\zeta}^{\dot{C}}}^{\text{=0}} + \alpha \mathbb{L}_{BA}^\mu \overbrace{\zeta^B \partial_\mu \star \bar{\zeta}^{\dot{C}}}_{\text{=0}} = \delta^{\dot{C}}_{\dot{A}} \end{aligned}$$

$$\partial_A \star \zeta^B \partial_\nu = \delta^B \partial_\nu$$

$$\bar{\zeta}^{\dot{B}} \partial_\mu \star \bar{\partial}_{\dot{A}} = \delta^{\dot{B}} \partial_\mu$$

$$\bar{\zeta}^{\dot{B}} \partial_\mu \star \zeta^B \partial_\nu = 0$$

$$\partial_A \star \zeta^B \partial_\nu = \partial_A \zeta^B \partial_\nu + \zeta^B \partial_\nu \partial_A = \partial_\nu \partial_A \zeta^B + \partial_\nu \zeta^B \partial_A = \partial_\nu \underbrace{\partial_A \zeta^B + \zeta^B \partial_A}_{= \delta^B} = \partial_\nu \underbrace{\partial_A \star \zeta^B}_{= \delta^B} = \delta^B \partial_\nu$$

$$\bar{\zeta}^{\dot{B}} \partial_\mu \star \bar{\partial}_{\dot{A}} = \bar{\zeta}^{\dot{B}} \partial_\mu \bar{\partial}_{\dot{A}} + \bar{\partial}_{\dot{A}} \bar{\zeta}^{\dot{B}} \partial_\mu = \bar{\zeta}^{\dot{B}} \bar{\partial}_{\dot{A}} \partial_\mu + \bar{\partial}_{\dot{A}} \bar{\zeta}^{\dot{B}} \partial_\mu = \underbrace{\bar{\zeta}^{\dot{B}} \bar{\partial}_{\dot{A}} + \bar{\partial}_{\dot{A}} \bar{\zeta}^{\dot{B}}}_{= \delta^{\dot{B}}} \partial_\mu = \bar{\partial}_{\dot{A}} \bar{\zeta}^{\dot{B}} \partial_\mu = \delta^{\dot{B}} \partial_\mu$$

$$\bar{\zeta}^{\dot{B}} \partial_\mu \star \zeta^B \partial_\nu = \bar{\zeta}^{\dot{B}} \partial_\mu \zeta^B \partial_\nu + \zeta^B \partial_\nu \bar{\zeta}^{\dot{B}} \partial_\mu = \bar{\zeta}^{\dot{B}} \zeta^B \partial_\mu \partial_\nu + \zeta^B \bar{\zeta}^{\dot{B}} \partial_\nu \partial_\mu = \underbrace{\bar{\zeta}^{\dot{B}} \zeta^B + \zeta^B \bar{\zeta}^{\dot{B}}}_{= 0} \partial_\nu \partial_\mu = \underbrace{\zeta^B \star \bar{\zeta}^{\dot{B}}}_{= 0} \partial_\nu \partial_\mu = 0$$

$$Q_A \star \bar{Q}_{\dot{A}} = -2 \mathbb{L}^{\mu}_{AA} \partial_\mu = -D_A \star \bar{D}_{\dot{A}}$$

$$D_A \star \bar{Q}_{\dot{A}} = 0 = Q_A \star \bar{D}_{\dot{A}}$$

$$\partial_A + \alpha \mathbb{L}^{\mu}_{AB} \bar{\zeta}^{\dot{B}} \partial_\mu \star \bar{\partial}_{\dot{A}} + \beta \zeta^B \mathbb{L}^{\nu}_{BA} \partial_\nu = \underbrace{\partial_A \star \bar{\partial}_{\dot{A}}}_{= 0} + \beta \underbrace{\partial_A \star \zeta^B \partial_\nu}_{= \delta^B \partial_\nu} \mathbb{L}^{\nu}_{BA}$$

$$+ \alpha \mathbb{L}^{\mu}_{AB} \underbrace{\bar{\zeta}^{\dot{B}} \partial_\mu \star \bar{\partial}_{\dot{A}}}_{= \delta^{\dot{B}} \partial_\mu} + \alpha \beta \mathbb{L}^{\mu}_{AB} \underbrace{\bar{\zeta}^{\dot{B}} \partial_\mu \star \zeta^B \partial_\nu}_{= 0} \mathbb{L}^{\nu}_{BA} = \beta \mathbb{L}^{\nu}_{AA} \partial_\nu + \alpha \mathbb{L}^{\mu}_{AA} \partial_\mu = \underbrace{\alpha + \beta}_{= 1} \mathbb{L}^{\mu}_{AA} \partial_\mu$$

$$\text{rep } [x:\xi] = \underline{x\mathbb{L}^\mu} \partial_\mu + \xi^A Q_A + \bar{\xi}^{\dot{A}} \bar{Q}_{\dot{A}}$$

$$\begin{aligned} [x:\xi] * [y:\eta] &= \overbrace{\underline{x\mathbb{L}^\mu} \partial_\mu + \xi^A Q_A + \bar{\xi}^{\dot{A}} \bar{Q}_{\dot{A}}} * \overbrace{\underline{y\mathbb{L}^\nu} \partial_\nu + \eta^B Q_B + \bar{\eta}^{\dot{B}} \bar{Q}_{\dot{B}}} \\ &= \underline{x\mathbb{L}^\mu} \partial_\mu * \underline{y\mathbb{L}^\nu} \partial_\nu + \underline{x\mathbb{L}^\mu} \partial_\mu * \eta^B Q_B + \underline{x\mathbb{L}^\mu} \partial_\mu * \bar{\eta}^{\dot{B}} \bar{Q}_{\dot{B}} + \end{aligned}$$

$$\xi^A Q_A * \underline{y\mathbb{L}^\nu} \partial_\nu + \xi^A Q_A * \eta^B Q_B + \xi^A Q_A * \bar{\eta}^{\dot{B}} \bar{Q}_{\dot{B}} + \bar{\xi}^{\dot{A}} \bar{Q}_{\dot{A}} * \underline{y\mathbb{L}^\nu} \partial_\nu + \bar{\xi}^{\dot{A}} \bar{Q}_{\dot{A}} * \eta^B Q_B + \bar{\xi}^{\dot{A}} \bar{Q}_{\dot{A}} * \bar{\eta}^{\dot{B}} \bar{Q}_{\dot{B}}$$

$$= \underline{x\mathbb{L}^\mu} \underline{y\mathbb{L}^\nu} \overbrace{\partial_\mu * \partial_\nu}^{=0} + \underline{x\mathbb{L}^\mu} \eta^B \overbrace{\partial_\mu * Q_B}^{=0} + \underline{x\mathbb{L}^\mu} \bar{\eta}^{\dot{B}} \overbrace{\partial_\mu * \bar{Q}_{\dot{B}}}^{=0} +$$

$$\xi^A \underline{y\mathbb{L}^\nu} \overbrace{Q_A * \partial_\nu}^{=0} + \xi^A \eta^B \overbrace{Q_A * Q_B}^{=0} + \xi^A \bar{\eta}^{\dot{B}} \overbrace{Q_A * \bar{Q}_{\dot{B}}}^{=0} + \bar{\xi}^{\dot{A}} \underline{y\mathbb{L}^\nu} \overbrace{\bar{Q}_{\dot{A}} * \partial_\nu}^{=0} + \bar{\xi}^{\dot{A}} \eta^B \overbrace{\bar{Q}_{\dot{A}} * Q_B}^{=0} + \bar{\xi}^{\dot{A}} \bar{\eta}^{\dot{B}} \overbrace{\bar{Q}_{\dot{A}} * \bar{Q}_{\dot{B}}}^{=0}$$

$$= -2 \underbrace{\xi^A \bar{\eta}^{\dot{B}} \mathbb{L}^\mu_{\dot{A}\dot{B}} \partial_\mu + \bar{\xi}^{\dot{A}} \eta^B \mathbb{L}^\mu_{\dot{B}A} \partial_\mu}_{= -2 \mathbb{L}^\mu_{\dot{A}\dot{B}} \partial_\mu} = -2 \underbrace{\xi^A \bar{\eta}^{\dot{B}} + \bar{\xi}^{\dot{B}} \eta^A}_{= -2 \mathbb{L}^\mu_{\dot{A}\dot{B}} \partial_\mu} \mathbb{L}^\mu_{\dot{A}\dot{B}} \partial_\mu$$

$$\begin{aligned} \overset{*}{\xi} \eta + \overset{*}{\eta} \xi &= \begin{bmatrix} \bar{\xi}^1 \\ \bar{\xi}^2 \end{bmatrix} \begin{bmatrix} \eta^1 & \eta^2 \end{bmatrix} + \begin{bmatrix} \bar{\eta}^1 \\ \bar{\eta}^2 \end{bmatrix} \begin{bmatrix} \xi^1 & \xi^2 \end{bmatrix} = \frac{\bar{\xi}^1 \eta^1 + \bar{\eta}^1 \xi^1}{\bar{\xi}^2 \eta^1 + \bar{\eta}^2 \xi^1} \left| \frac{\bar{\xi}^1 \eta^2 + \bar{\eta}^1 \xi^2}{\bar{\xi}^2 \eta^2 + \bar{\eta}^2 \xi^2} \right. \end{aligned}$$

$$[\overset{*}{\xi} \eta + \overset{*}{\eta} \xi] = \underline{\mathbb{L}^\mu \overset{*}{\xi} \eta + \overset{*}{\eta} \xi} \partial_\mu = \overset{A}{\mathbb{L}^\mu \overset{*}{\xi} \eta + \overset{*}{\eta} \xi} \partial_\mu = \overset{A}{\mathbb{L}^\mu_{\dot{B}}} \overset{\dot{B}}{\overset{*}{\xi} \eta + \overset{*}{\eta} \xi} \partial_\mu$$

$$= \overset{A}{\mathbb{L}^\mu_{\dot{B}}} \underbrace{\bar{\xi}^{\dot{B}} \eta^A + \bar{\eta}^{\dot{B}} \xi^A}_{= \xi^{AA} \mathbb{L}^\mu_{\dot{B}} \bar{\eta}^{\dot{B}} + \eta^{AA} \mathbb{L}^\mu_{\dot{B}} \bar{\xi}^{\dot{B}}} \partial_\mu = \xi^{AA} \mathbb{L}^\mu_{\dot{B}} \bar{\eta}^{\dot{B}} + \eta^{AA} \mathbb{L}^\mu_{\dot{B}} \bar{\xi}^{\dot{B}} \partial_\mu$$