$$\text{dof } {}^{i}_{\sigma}\nvDash_{\ell} : {}^{i}_{\mu\sigma}\blacktriangleright_{\ell} \in {}^{N}_{d}\mathbb{K}_{N} \times {}^{N}_{dd}\mathbb{K}_{N}$$

$$x^{\nu} : \nvDash : \blacktriangleright \in \mathbb{R}^{d} \times {}^{N}_{d}\mathbb{R}_{N} \times {}^{N}_{d}\mathbb{R}_{N} \xrightarrow[\text{Yang-Mills}]{\mathcal{L}} \mathbb{R} \ni {}^{x}\mathcal{L}_{\nvDash : \blacktriangleright}$$

$${}^{x}\mathcal{L}_{\nvDash : \blacktriangleright} = tr \overbrace{{}_{\alpha\beta}\dot{\blacktriangleright}_{\cdot} - {}_{\beta\alpha}\dot{\blacktriangleright}_{\cdot} + {}_{\alpha}\dot{\nvDash}_{\cdot\,\beta}\dot{\nvDash}_{\cdot} - {}_{\beta}\dot{\nvDash}_{\cdot\,\alpha}\dot{\nvDash}_{\cdot}}^{+} \eta^{\alpha\mu}\,\eta^{\beta\nu} \overbrace{{}_{\mu\nu}\dot{\blacktriangleright}_{\cdot} - {}_{\nu\mu}\dot{\blacktriangleright}_{\cdot} + {}_{\mu}\dot{\nvDash}_{\cdot\,\nu}\dot{\nvDash}_{\cdot} - {}_{\nu}\dot{\nvDash}_{\cdot\,\mu}\dot{\nvDash}_{\cdot}}$$

$$= \underbrace{{}^{j}_{\alpha\beta}\blacktriangleright_{i} - {}^{j}_{\beta\alpha}\blacktriangleright_{i} + {}^{j}_{\alpha}\nvDash_{\ell\,\beta}\nvDash^{\ell}_{i} - {}^{j}_{\beta}\nvDash_{\ell\,\alpha}\nvDash^{\ell}_{i}}\,\eta^{\alpha\mu}\,\eta^{\beta\nu}\,\underbrace{{}^{i}_{\mu\nu}\blacktriangleright_{j} - {}^{i}_{\nu\mu}\blacktriangleright_{j} + {}^{i}_{\mu}\nvDash^{k}_{\nu}\nvDash_{kj} - {}^{i}_{\nu}\nvDash^{k}_{\mu}\nvDash_{kj}}$$

$$\text{fields } \mathbb{R}^{d} \xrightarrow{{}^{i}_{\sigma}\nvDash_{\ell}} {}^{N}_{d}\mathbb{R}_{N} \ni {}^{x}{}^{i}_{\sigma}\nvDash_{\ell}$$

$$\frac{1}{4}\,\underbrace{\mathcal{L}^{n}\partial^{\tau}_{m}}_{\text{symmetrized}} = \frac{1}{4}\frac{\partial\mathcal{L}}{\partial\,{}^{m}_{\tau}\Gamma^{n}_{\beta}} = \eta^{\tau\mu}\,\eta^{\beta\nu}\left({}_{\beta}\Gamma\ast{}^{n}_{\mu\nu}F\right)_{m} = \eta^{\tau\mu}\,\eta^{\beta\nu}\left({}_{\beta}\Gamma_{\mu\nu}F - {}^{n}_{\mu\nu}F\,{}_{\beta}\Gamma\right)_{m} = \eta^{\tau\mu}\,\eta^{\beta\nu}\left({}^{n}_{\beta}\Gamma_{i\mu\nu}\,{}^{i}F_{m} - {}^{n}_{\mu\nu}F_{j}\,{}^{j}_{\beta}\Gamma_{m}\right)$$

$$= \eta^{\tau\mu}\,\eta^{\sigma\nu}\,\overbrace{{}^{n}_{\sigma}\Gamma_{i}\underbrace{{}^{i}_{\mu\nu}\Gamma_{m} - {}^{i}_{\nu\mu}\Gamma_{m} + {}^{i}_{\mu}{}_{k}\,{}^{k}_{\nu}\Gamma_{m} - {}^{i}_{\nu}{}_{k}\,{}^{k}_{\mu}\Gamma_{m}}_{\text{symmetrized}} - \underbrace{{}^{n}_{\mu\nu}\Gamma_{j} - {}^{n}_{\nu\mu}\Gamma_{j} + {}^{n}_{\mu}{}_{k}\,{}^{k}_{\nu}\Gamma_{j} - {}^{n}_{\nu}{}_{k}\,{}^{k}_{\mu}\Gamma_{j}}_{\text{symmetrized}}\,{}^{j}_{\sigma}\Gamma_{m}}}$$

$$\frac{\partial\mathcal{L}}{\partial\,{}^{m}_{\tau}\Gamma^{n}_{\beta}} =$$

$$\begin{aligned}
&{}^{n}_{\beta}\Gamma_{i}\,\eta^{\tau\mu}\,\eta^{\beta\nu}\underbrace{{}^{i}_{\mu\nu}\Gamma_{m} - {}^{i}_{\nu\mu}\Gamma_{m} + {}^{i}_{\mu}{}_{k}\,{}^{k}_{\nu}\Gamma_{m} - {}^{i}_{\nu}{}_{k}\,{}^{k}_{\mu}\Gamma_{m}}_{j=m:\alpha=\tau:\ell=n} + {}^{j}_{\alpha}\Gamma_{m}\,\eta^{\alpha\mu}\,\eta^{\tau\nu}\underbrace{{}^{n}_{\mu\nu}\Gamma_{j} - {}^{n}_{\nu\mu}\Gamma_{j} + {}^{n}_{\mu}{}_{k}\,{}^{k}_{\nu}\Gamma_{j} - {}^{n}_{\nu}{}_{k}\,{}^{k}_{\mu}\Gamma_{j}}_{\ell=m:\beta=\tau:i=n}\\
&- {}^{n}_{\alpha}\Gamma_{i}\,\eta^{\alpha\mu}\,\eta^{\tau\nu}\underbrace{{}^{i}_{\mu\nu}\Gamma_{m} - {}^{i}_{\nu\mu}\Gamma_{m} + {}^{i}_{\mu}{}_{k}\,{}^{k}_{\nu}\Gamma_{m} - {}^{i}_{\nu}{}_{k}\,{}^{k}_{\mu}\Gamma_{m}}_{j=m:\beta=\tau:\ell=n} - {}^{j}_{\beta}\Gamma_{m}\,\eta^{\tau\mu}\,\eta^{\beta\nu}\underbrace{{}^{n}_{\mu\nu}\Gamma_{j} - {}^{n}_{\nu\mu}\Gamma_{j} + {}^{n}_{\mu}{}_{k}\,{}^{k}_{\nu}\Gamma_{j} - {}^{n}_{\nu}{}_{k}\,{}^{k}_{\mu}\Gamma_{j}}_{\ell=m:\alpha=\tau:i=n}\\
&+ \underbrace{{}^{j}_{\alpha\beta}\Gamma_{m} - {}^{j}_{\beta\alpha}\Gamma_{m} + {}^{j}_{\alpha}{}_{\ell}\,{}^{\ell}_{\beta}\Gamma_{m} - {}^{j}_{\beta}{}_{\ell}\,{}^{\ell}_{\alpha}\Gamma_{m}}_{i=m:\mu=\tau:k=n}\,\eta^{\alpha\tau}\,\eta^{\beta\nu}\,{}^{n}_{\nu}\Gamma_{j} + \underbrace{{}^{n}_{\alpha\beta}\Gamma_{i} - {}^{n}_{\beta\alpha}\Gamma_{i} + {}^{n}_{\alpha}{}_{\ell}\,{}^{\ell}_{\beta}\Gamma_{i} - {}^{n}_{\beta}{}_{\ell}\,{}^{\ell}_{\alpha}\Gamma_{i}}_{k=m:\nu=\tau:j=n}\,\eta^{\alpha\mu}\,\eta^{\beta\tau}\,{}^{i}_{\mu}\Gamma_{m}\\
&- \underbrace{{}^{j}_{\alpha\beta}\Gamma_{m} - {}^{j}_{\beta\alpha}\Gamma_{m} + {}^{j}_{\alpha}{}_{\ell}\,{}^{\ell}_{\beta}\Gamma_{m} - {}^{j}_{\beta}{}_{\ell}\,{}^{\ell}_{\alpha}\Gamma_{m}}_{i=m:\nu=\tau:k=n}\,\eta^{\alpha\mu}\,\eta^{\beta\tau}\,{}^{n}_{\mu}\Gamma_{j} - \underbrace{{}^{n}_{\alpha\beta}\Gamma_{i} - {}^{n}_{\beta\alpha}\Gamma_{i} + {}^{n}_{\alpha}{}_{\ell}\,{}^{\ell}_{\beta}\Gamma_{i} - {}^{n}_{\beta}{}_{\ell}\,{}^{\ell}_{\alpha}\Gamma_{i}}_{k=m:\mu=\tau:j=n}\,\eta^{\alpha\tau}\,\eta^{\beta\nu}\,{}^{i}_{\nu}\Gamma_{m}\\
&= {}^{n}_{\beta}\Gamma_{i}\,\eta^{\tau\mu}\,\eta^{\beta\nu}\,{}^{i}_{\mu\nu}F_{m} + {}^{j}_{\alpha}\Gamma_{m}\,\eta^{\alpha\mu}\,\eta^{\tau\nu}\,{}^{n}_{\mu\nu}F_{j} - {}^{n}_{\alpha}\Gamma_{i}\,\eta^{\alpha\mu}\,\eta^{\tau\nu}\,{}^{i}_{\mu\nu}F_{m} - {}^{j}_{\beta}\Gamma_{m}\,\eta^{\tau\mu}\,\eta^{\beta\nu}\,{}^{n}_{\mu\nu}F_{j}\\
&+ {}^{j}_{\alpha\beta}F_{m}\,\eta^{\alpha\tau}\,\eta^{\beta\nu}\,{}^{n}_{\nu}\Gamma_{j} + {}^{n}_{\alpha\beta}F_{i}\,\eta^{\alpha\mu}\,\eta^{\beta\tau}\,{}^{i}_{\mu}\Gamma_{m} - {}^{j}_{\alpha\beta}F_{m}\,\eta^{\alpha\mu}\,\eta^{\beta\tau}\,{}^{n}_{\mu}\Gamma_{j} - {}^{n}_{\alpha\beta}F_{i}\,\eta^{\alpha\tau}\,\eta^{\beta\nu}\,{}^{i}_{\nu}\Gamma_{m}\\
&= \eta^{\tau\mu}\,\eta^{\beta\nu}\,{}^{n}_{\beta}\Gamma_{i}\,{}^{i}_{\mu\nu}F_{m} + \eta^{\alpha\mu}\,\eta^{\tau\nu}\,{}^{n}_{\mu\nu}F_{j}\,{}^{j}_{\alpha}\Gamma_{m} - \eta^{\alpha\mu}\,\eta^{\tau\nu}\,{}^{n}_{\delta}\Gamma_{i}\,{}^{i}_{\mu\nu}I_{m} - \eta^{\tau\mu}\,\eta^{\beta\nu}\,{}^{n}_{\mu\nu}F_{j}\,{}^{j}_{\beta}\Gamma_{m}\\
&+ \eta^{\alpha\tau}\,\eta^{\beta\nu}\,{}^{n}_{\nu}\Gamma_{j}\,{}^{j}_{\alpha\beta}F_{m} + \eta^{\alpha\mu}\,\eta^{\beta\tau}\,{}^{n}_{\alpha\beta}F_{i}\,{}^{i}_{\mu}\Gamma_{m} - \eta^{\alpha\mu}\,\eta^{\beta\tau}\,{}^{n}_{\mu}\Gamma_{j}\,{}^{j}_{\alpha\beta}F_{m} - \eta^{\alpha\tau}\,\eta^{\beta\nu}\,{}^{n}_{\alpha\beta}F_{i}\,{}^{i}_{\nu}\Gamma_{m}\\
&= \eta^{\tau\mu}\,\eta^{\beta\nu}\,{}^{n}_{\beta}\Gamma_{i}\,{}^{i}_{\mu\nu}F_{m} - \eta^{\alpha\mu}\,\eta^{\tau\nu}\,{}^{n}_{\nu\mu}F_{j}\,{}^{j}_{\alpha}\Gamma_{m} + \eta^{\alpha\mu}\,\eta^{\tau\nu}\,{}^{n}_{\delta}\Gamma_{i}\,{}^{i}_{\nu\mu}I_{m} - \eta^{\tau\mu}\,\eta^{\beta\nu}\,{}^{n}_{\mu\nu}F_{j}\,{}^{j}_{\beta}\Gamma_{m}\\
&+ \eta^{\alpha\tau}\,\eta^{\beta\nu}\,{}^{n}_{\nu}\Gamma_{j}\,{}^{j}_{\alpha\beta}F_{m} - \eta^{\alpha\mu}\,\eta^{\beta\tau}\,{}^{n}_{\beta\alpha}F_{i}\,{}^{i}_{\mu}\Gamma_{m} + \eta^{\alpha\mu}\,\eta^{\beta\tau}\,{}^{n}_{\mu}\Gamma_{j}\,{}^{j}_{\beta\alpha}F_{m} - \eta^{\alpha\tau}\,\eta^{\beta\nu}\,{}^{n}_{\alpha\beta}F_{i}\,{}^{i}_{\nu}\Gamma_{m}\\
&= 4\,\eta^{\tau\mu}\,\eta^{\beta\nu}\left({}^{n}_{\beta}\Gamma_{i\mu\nu}\,{}^{i}F_{m} - {}^{n}_{\mu\nu}F_{j}\,{}^{j}_{\beta}\Gamma_{m}\right)
\end{aligned}$$

$$\frac{1}{4}\frac{\partial\mathcal{L}}{\partial\,{}^{m}_{\sigma\tau}\blacktriangledown_{n}} = \eta^{\sigma\mu}\,\eta^{\tau\nu}\,{}^{n}_{\mu\nu}F_{m} = \eta^{\sigma\mu}\,\eta^{\tau\nu}\,\underbrace{{}^{n}_{\mu\nu}\blacktriangledown_{m} - {}^{n}_{\nu\mu}\blacktriangledown_{m} + {}^{n}_{\mu}{}^{k}_{\nu}\blacktriangledown_{m} - {}^{n}_{\nu}{}^{k}_{\mu}\blacktriangledown_{m}}$$

$$\frac{\partial\mathcal{L}}{\partial\,{}^{m}_{\sigma\tau}\blacktriangledown_{n}} = \eta^{\sigma\mu}\,\eta^{\tau\nu}\,\underbrace{{}^{n}_{\mu\nu}\blacktriangledown_{m} - {}^{n}_{\nu\mu}\blacktriangledown_{m} + {}^{n}_{\mu}{}^{k}_{\nu}\blacktriangledown_{m} - {}^{n}_{\nu}{}^{k}_{\mu}\blacktriangledown_{m}}_{j=m:i=n:\alpha=\sigma:\beta=\tau} - \eta^{\tau\mu}\,\eta^{\sigma\nu}\,\underbrace{{}^{n}_{\mu\nu}\blacktriangledown_{m} - {}^{n}_{\nu\mu}\blacktriangledown_{m} + {}^{n}_{\mu}{}^{k}_{\nu}\blacktriangledown_{m} - {}^{n}_{\nu}{}^{k}_{\mu}\blacktriangledown_{m}}_{j=m:i=n:\alpha=\tau:\beta=\sigma}$$

$$+ \underbrace{{}^{n}_{\alpha\beta}\blacktriangledown_{m} - {}^{n}_{\beta\alpha}\blacktriangledown_{m} + {}^{n}_{\alpha}{}^{\ell}_{\beta}\blacktriangledown_{m} - {}^{n}_{\beta}{}^{\ell}_{\alpha}\blacktriangledown_{m}}_{i=m:j=n:\mu=\sigma:\nu=\tau}\,\eta^{\alpha\sigma}\,\eta^{\beta\tau} - \underbrace{{}^{n}_{\alpha\beta}\blacktriangledown_{m} - {}^{n}_{\beta\alpha}\blacktriangledown_{m} + {}^{n}_{\alpha}{}^{\ell}_{\beta}\blacktriangledown_{m} - {}^{n}_{\beta}{}^{\ell}_{\alpha}\blacktriangledown_{m}}_{i=m:j=n:\mu=\tau:\nu=\sigma}\,\eta^{\alpha\tau}\,\eta^{\beta\sigma}$$

$$= \eta^{\sigma\mu}\,\eta^{\tau\nu}\,{}^{n}_{\mu\nu}F_{m} - \eta^{\tau\mu}\,\eta^{\sigma\nu}\,{}^{n}_{\mu\nu}F_{m} + {}^{n}_{\alpha\beta}F_{m}\,\eta^{\alpha\sigma}\,\eta^{\beta\tau} - {}^{n}_{\alpha\beta}F_{m}\,\eta^{\alpha\tau}\,\eta^{\beta\sigma}$$

$$= \eta^{\sigma\mu}\,\eta^{\tau\nu}\,{}^{n}_{\mu\nu}F_{m} + \eta^{\sigma\nu}\,\eta^{\tau\mu}\,{}^{n}_{\nu\mu}F_{m} + {}^{n}_{\alpha\beta}F_{m}\,\eta^{\sigma\alpha}\,\eta^{\tau\beta} + {}^{n}_{\beta\alpha}F_{m}\,\eta^{\sigma\beta}\,\eta^{\tau\alpha} = 4\,\eta^{\sigma\mu}\,\eta^{\tau\nu}\,{}^{n}_{\mu\nu}F_{m}$$