

$$\text{dof } \mathbf{\Psi}_{\ell}^i \colon \mathbf{\Psi}_{\ell}^i \in {}^N_d\mathbb{K}_N \times {}^N_{dd}\mathbb{K}_N$$

$$x^\nu\colon\mathbf{\Psi}\colon\mathbf{\Psi}\in\mathbb{R}^d\times^N\mathbb{R}_N\times^N_d\mathbb{R}_N\overset{\mathcal{L}}{\underset{\text{Yang-Mills}}{\longrightarrow}}\mathbb{R}\ni{}^x\mathcal{L}_{\mathbf{\Psi}\colon\mathbf{\Psi}}$$

$$\begin{aligned} {}^x\mathcal{L}_{\mathbf{\Psi}\colon\mathbf{\Psi}} &= tr\overbrace{\mathbf{\Psi}_{\alpha\beta}^i-\mathbf{\Psi}_{\beta\alpha}^i+\mathbf{\Psi}_{\alpha}^i\mathbf{\Psi}_{\beta}^i-\mathbf{\Psi}_{\beta}^i\mathbf{\Psi}_{\alpha}^i}^+\eta^{\alpha\mu}\eta^{\beta\nu}\overbrace{\mathbf{\Psi}_{\mu\nu}^i-\mathbf{\Psi}_{\nu\mu}^i+\mathbf{\Psi}_{\mu}^i\mathbf{\Psi}_{\nu}^i-\mathbf{\Psi}_{\nu}^i\mathbf{\Psi}_{\mu}^i}^+\\ &= \underbrace{\mathbf{\Psi}_{\alpha\beta}^j-\mathbf{\Psi}_{\beta\alpha}^j+\mathbf{\Psi}_{\ell\beta}^j\mathbf{\Psi}_i^{\ell}-\mathbf{\Psi}_{\ell\alpha}^j\mathbf{\Psi}_i^{\ell}}_{}\eta^{\alpha\mu}\eta^{\beta\nu}\underbrace{\mathbf{\Psi}_{\mu\nu}^i-\mathbf{\Psi}_{\nu\mu}^i+\mathbf{\Psi}_{\mu}^i\mathbf{\Psi}_k^{\nu}\mathbf{\Psi}_j^k-\mathbf{\Psi}_{\nu}^i\mathbf{\Psi}_k^{\mu}\mathbf{\Psi}_j^k}_{} \end{aligned}$$

$$\text{fields } \mathbb{R}^d \overset{\mathbf{\Psi}_{\ell}^i}{\longrightarrow} {}^N_d\mathbb{R}_N \ni {}^x_{\mathbf{\Psi}_{\ell}^i}$$

$$\begin{aligned}
\frac{1}{4} \textcolor{violet}{x} \underbrace{\mathcal{L}^n \partial_m^\tau}_{\Psi:\Psi} &= \frac{1}{4} \frac{\partial \mathcal{L}}{\partial \tau \textcolor{violet}{\Psi}_n^m} = \eta^{\tau\mu} \eta^{\beta\nu} \left(\textcolor{violet}{\Psi}^n \textcolor{violet}{\times}_{\mu\nu} F \right)_m = \eta^{\tau\mu} \eta^{\beta\nu} \left(\textcolor{violet}{\Psi}_{\mu\nu} F \textcolor{violet}{-}^n_{\mu\nu} F \textcolor{violet}{\Psi}_\beta \right)_m = \eta^{\tau\mu} \eta^{\beta\nu} \left(\textcolor{violet}{\Psi}_{\beta i \mu\nu}^i F_m - \textcolor{violet}{\Psi}_{\mu\nu}^n F_j \textcolor{violet}{\Psi}_{j\beta}^j m \right) \\
&= \eta^{\tau\mu} \eta^{\sigma\nu} \textcolor{violet}{\Psi}_{\sigma \textcolor{violet}{\mu} \nu}^n \underbrace{\textcolor{violet}{\Psi}_{i \mu\nu}^i m - \textcolor{violet}{\Psi}_{\nu\mu}^i m + \textcolor{violet}{\Psi}_{\mu}^i \textcolor{violet}{\Psi}_{k\nu}^k m - \textcolor{violet}{\Psi}_{\nu}^i \textcolor{violet}{\Psi}_{k\mu}^k m}_{j=m:\alpha=\tau:\ell=n} - \underbrace{\textcolor{violet}{\Psi}_{\mu\nu}^n j - \textcolor{violet}{\Psi}_{\nu\mu}^n j + \textcolor{violet}{\Psi}_{\mu}^n \textcolor{violet}{\Psi}_{k\nu}^k j - \textcolor{violet}{\Psi}_{\nu}^n \textcolor{violet}{\Psi}_{k\mu}^k j}_{\ell=m:\beta=\tau:i=n} \textcolor{violet}{\Psi}_\sigma^j m
\end{aligned}$$

$$\begin{aligned}
&\frac{\partial \mathcal{L}}{\partial \tau \textcolor{violet}{\Psi}_n^m} = \\
&\textcolor{violet}{\Psi}_\beta^i \eta^{\tau\mu} \eta^{\beta\nu} \underbrace{\textcolor{violet}{\Psi}_{\mu\nu}^i m - \textcolor{violet}{\Psi}_{\nu\mu}^i m + \textcolor{violet}{\Psi}_{\mu}^i \textcolor{violet}{\Psi}_{k\nu}^k m - \textcolor{violet}{\Psi}_{\nu}^i \textcolor{violet}{\Psi}_{k\mu}^k m}_{j=m:\alpha=\tau:\ell=n} + \textcolor{violet}{\Psi}_m^j \eta^{\alpha\mu} \eta^{\tau\nu} \underbrace{\textcolor{violet}{\Psi}_{\mu\nu}^n j - \textcolor{violet}{\Psi}_{\nu\mu}^n j + \textcolor{violet}{\Psi}_{\mu}^n \textcolor{violet}{\Psi}_{k\nu}^k j - \textcolor{violet}{\Psi}_{\nu}^n \textcolor{violet}{\Psi}_{k\mu}^k j}_{\ell=m:\beta=\tau:i=n} \\
&- \textcolor{violet}{\Psi}_i^{\textcolor{violet}{\Psi}} \eta^{\alpha\mu} \eta^{\tau\nu} \underbrace{\textcolor{violet}{\Psi}_{\mu\nu}^i m - \textcolor{violet}{\Psi}_{\nu\mu}^i m + \textcolor{violet}{\Psi}_{\mu}^i \textcolor{violet}{\Psi}_{k\nu}^k m - \textcolor{violet}{\Psi}_{\nu}^i \textcolor{violet}{\Psi}_{k\mu}^k m}_{j=m:\beta=\tau:\ell=n} - \textcolor{violet}{\Psi}_m^j \eta^{\tau\mu} \eta^{\beta\nu} \underbrace{\textcolor{violet}{\Psi}_{\mu\nu}^n j - \textcolor{violet}{\Psi}_{\nu\mu}^n j + \textcolor{violet}{\Psi}_{\mu}^n \textcolor{violet}{\Psi}_{k\nu}^k j - \textcolor{violet}{\Psi}_{\nu}^n \textcolor{violet}{\Psi}_{k\mu}^k j}_{\ell=m:\alpha=\tau:i=n} \\
&+ \underbrace{\textcolor{violet}{\Psi}_m^j - \textcolor{violet}{\Psi}_\beta^j m + \textcolor{violet}{\Psi}_\alpha^j \textcolor{violet}{\Psi}_{\ell\beta}^\ell m - \textcolor{violet}{\Psi}_\beta^j \textcolor{violet}{\Psi}_{\ell\alpha}^\ell m}_{i=m:\mu=\tau:k=n} \eta^{\alpha\tau} \eta^{\beta\nu} \textcolor{violet}{\Psi}_\nu^{\textcolor{violet}{\Psi}} j + \underbrace{\textcolor{violet}{\Psi}_i^{\textcolor{violet}{\Psi}} - \textcolor{violet}{\Psi}_\beta^{\textcolor{violet}{\Psi}} m + \textcolor{violet}{\Psi}_\alpha^{\textcolor{violet}{\Psi}} \textcolor{violet}{\Psi}_{\ell\beta}^\ell i - \textcolor{violet}{\Psi}_\beta^{\textcolor{violet}{\Psi}} \textcolor{violet}{\Psi}_{\ell\alpha}^\ell i}_{k=m:\nu=\tau:j=n} \eta^{\alpha\mu} \eta^{\beta\tau} \textcolor{violet}{\Psi}_\mu^i m \\
&- \underbrace{\textcolor{violet}{\Psi}_m^j - \textcolor{violet}{\Psi}_\beta^j m + \textcolor{violet}{\Psi}_\alpha^j \textcolor{violet}{\Psi}_{\ell\beta}^\ell m - \textcolor{violet}{\Psi}_\beta^j \textcolor{violet}{\Psi}_{\ell\alpha}^\ell m}_{i=m:\nu=\tau:k=n} \eta^{\alpha\mu} \eta^{\beta\tau} \textcolor{violet}{\Psi}_\mu^{\textcolor{violet}{\Psi}} j - \underbrace{\textcolor{violet}{\Psi}_i^{\textcolor{violet}{\Psi}} - \textcolor{violet}{\Psi}_\beta^{\textcolor{violet}{\Psi}} m + \textcolor{violet}{\Psi}_\alpha^{\textcolor{violet}{\Psi}} \textcolor{violet}{\Psi}_{\ell\beta}^\ell i - \textcolor{violet}{\Psi}_\beta^{\textcolor{violet}{\Psi}} \textcolor{violet}{\Psi}_{\ell\alpha}^\ell i}_{k=m:\mu=\tau:j=n} \eta^{\alpha\tau} \eta^{\beta\nu} \textcolor{violet}{\Psi}_\nu^i m \\
&= \textcolor{violet}{\Psi}_\beta^i \eta^{\tau\mu} \eta^{\beta\nu} \textcolor{violet}{\Psi}_{\mu\nu}^i F_m + \textcolor{violet}{\Psi}_m^j \eta^{\alpha\mu} \eta^{\tau\nu} \textcolor{violet}{\Psi}_{\mu\nu}^n F_j - \textcolor{violet}{\Psi}_i^{\textcolor{violet}{\Psi}} \eta^{\alpha\mu} \eta^{\tau\nu} \textcolor{violet}{\Psi}_{\mu\nu}^i F_m - \textcolor{violet}{\Psi}_m^j \eta^{\tau\mu} \eta^{\beta\nu} \textcolor{violet}{\Psi}_{\mu\nu}^n F_j \\
&+ \textcolor{violet}{\Psi}_\alpha^j F_m \eta^{\alpha\tau} \eta^{\beta\nu} \textcolor{violet}{\Psi}_\nu^{\textcolor{violet}{\Psi}} j + \textcolor{violet}{\Psi}_\beta^{\textcolor{violet}{\Psi}} F_i \eta^{\alpha\mu} \eta^{\beta\tau} \textcolor{violet}{\Psi}_\mu^i m - \textcolor{violet}{\Psi}_\alpha^j F_m \eta^{\alpha\mu} \eta^{\beta\tau} \textcolor{violet}{\Psi}_\mu^{\textcolor{violet}{\Psi}} j - \textcolor{violet}{\Psi}_\beta^{\textcolor{violet}{\Psi}} F_i \eta^{\alpha\tau} \eta^{\beta\nu} \textcolor{violet}{\Psi}_\nu^i m \\
&= \eta^{\tau\mu} \eta^{\beta\nu} \textcolor{violet}{\Psi}_{\beta i \mu\nu}^i F_m + \eta^{\alpha\mu} \eta^{\tau\nu} \textcolor{violet}{\Psi}_{\mu\nu}^n F_j \textcolor{violet}{\Psi}_\alpha^j m - \eta^{\alpha\mu} \eta^{\tau\nu} \textcolor{violet}{\Psi}_{\delta i \mu\nu}^i I_m - \eta^{\tau\mu} \eta^{\beta\nu} \textcolor{violet}{\Psi}_{\mu\nu}^n F_j \textcolor{violet}{\Psi}_\beta^j m \\
&+ \eta^{\alpha\tau} \eta^{\beta\nu} \textcolor{violet}{\Psi}_\nu^{\textcolor{violet}{\Psi}} j \textcolor{violet}{\Psi}_{\alpha\beta}^j F_m + \eta^{\alpha\mu} \eta^{\beta\tau} \textcolor{violet}{\Psi}_{\alpha\beta}^n F_i \textcolor{violet}{\Psi}_\mu^i m - \eta^{\alpha\mu} \eta^{\beta\tau} \textcolor{violet}{\Psi}_\mu^{\textcolor{violet}{\Psi}} j \textcolor{violet}{\Psi}_{\alpha\beta}^j F_m - \eta^{\alpha\tau} \eta^{\beta\nu} \textcolor{violet}{\Psi}_{\alpha\beta}^n F_i \textcolor{violet}{\Psi}_\nu^i m \\
&= \eta^{\tau\mu} \eta^{\beta\nu} \textcolor{violet}{\Psi}_{\beta i \mu\nu}^i F_m - \eta^{\alpha\mu} \eta^{\tau\nu} \textcolor{violet}{\Psi}_{\nu\mu}^n F_j \textcolor{violet}{\Psi}_\alpha^j m + \eta^{\alpha\mu} \eta^{\tau\nu} \textcolor{violet}{\Psi}_{\delta i \nu\mu}^i I_m - \eta^{\tau\mu} \eta^{\beta\nu} \textcolor{violet}{\Psi}_{\mu\nu}^n F_j \textcolor{violet}{\Psi}_\beta^j m \\
&+ \eta^{\alpha\tau} \eta^{\beta\nu} \textcolor{violet}{\Psi}_\nu^{\textcolor{violet}{\Psi}} j \textcolor{violet}{\Psi}_{\alpha\beta}^j F_m - \eta^{\alpha\mu} \eta^{\beta\tau} \textcolor{violet}{\Psi}_{\beta\alpha}^n F_i \textcolor{violet}{\Psi}_\mu^i m + \eta^{\alpha\mu} \eta^{\beta\tau} \textcolor{violet}{\Psi}_\mu^{\textcolor{violet}{\Psi}} j \textcolor{violet}{\Psi}_{\beta\alpha}^j F_m - \eta^{\alpha\tau} \eta^{\beta\nu} \textcolor{violet}{\Psi}_{\alpha\beta}^n F_i \textcolor{violet}{\Psi}_\nu^i m \\
&= 4 \eta^{\tau\mu} \eta^{\beta\nu} \left(\textcolor{violet}{\Psi}_{\beta i \mu\nu}^i F_m - \textcolor{violet}{\Psi}_{\mu\nu}^n F_j \textcolor{violet}{\Psi}_{j\beta}^j m \right)
\end{aligned}$$

$$\frac{1}{4} \frac{\partial \mathcal{L}}{\partial \sigma_{\tau}^{\mathfrak{N}}_n} = \eta^{\sigma\mu} \eta^{\tau\nu} {}^nF_{\mu\nu} = \eta^{\sigma\mu} \eta^{\tau\nu} \underbrace{{}_{\mu\nu}^{\mathfrak{N}}_m - {}_{\nu\mu}^{\mathfrak{N}}_m + {}_{\mu}^{\mathfrak{N}}_k {}_{\nu}^{\mathfrak{N}}_m - {}_{\nu}^{\mathfrak{N}}_k {}_{\mu}^{\mathfrak{N}}_m}$$

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \sigma_{\tau}^{\mathfrak{N}}_n} &= \eta^{\sigma\mu} \eta^{\tau\nu} \underbrace{{}_{\mu\nu}^{\mathfrak{N}}_m - {}_{\nu\mu}^{\mathfrak{N}}_m + {}_{\mu}^{\mathfrak{N}}_k {}_{\nu}^{\mathfrak{N}}_m - {}_{\nu}^{\mathfrak{N}}_k {}_{\mu}^{\mathfrak{N}}_m}_{j=m:i=n:\alpha=\sigma:\beta=\tau} - \eta^{\tau\mu} \eta^{\sigma\nu} \underbrace{{}_{\mu\nu}^{\mathfrak{N}}_m - {}_{\nu\mu}^{\mathfrak{N}}_m + {}_{\mu}^{\mathfrak{N}}_k {}_{\nu}^{\mathfrak{N}}_m - {}_{\nu}^{\mathfrak{N}}_k {}_{\mu}^{\mathfrak{N}}_m}_{j=m:i=n:\alpha=\tau:\beta=\sigma} \\ &+ \underbrace{{}_{\alpha\beta}^{\mathfrak{N}}_m - {}_{\beta\alpha}^{\mathfrak{N}}_m + {}_{\alpha}^{\mathfrak{N}}_{\ell} {}_{\beta}^{\ell} - {}_{\beta}^{\mathfrak{N}}_{\ell} {}_{\alpha}^{\ell}}_{i=m:j=n:\mu=\sigma:\nu=\tau} \eta^{\alpha\sigma} \eta^{\beta\tau} - \underbrace{{}_{\alpha\beta}^{\mathfrak{N}}_m - {}_{\beta\alpha}^{\mathfrak{N}}_m + {}_{\alpha}^{\mathfrak{N}}_{\ell} {}_{\beta}^{\ell} - {}_{\beta}^{\mathfrak{N}}_{\ell} {}_{\alpha}^{\ell}}_{i=m:j=n:\mu=\tau:\nu=\sigma} \eta^{\alpha\tau} \eta^{\beta\sigma} \\ &= \eta^{\sigma\mu} \eta^{\tau\nu} {}^nF_{\mu\nu} - \eta^{\tau\mu} \eta^{\sigma\nu} {}^nF_{\mu\nu} + {}^nF_m \eta^{\alpha\sigma} \eta^{\beta\tau} - {}^nF_m \eta^{\alpha\tau} \eta^{\beta\sigma} \\ &= \eta^{\sigma\mu} \eta^{\tau\nu} {}^nF_{\mu\nu} + \eta^{\sigma\nu} \eta^{\tau\mu} {}^nF_{\nu\mu} + {}^nF_m \eta^{\sigma\alpha} \eta^{\tau\beta} + {}^nF_m \eta^{\sigma\beta} \eta^{\tau\alpha} = 4 \eta^{\sigma\mu} \eta^{\tau\nu} {}^nF_{\mu\nu} \end{aligned}$$