$${}^{x}\underbrace{\partial_\mu \mathcal{L}}_{\Upsilon:\Upsilon} = 0$$

$${}^{x}\underbrace{\mathcal{L}^n\partial^\sigma_m}_{\Upsilon} = 4\underbrace{{}_{\mu\nu}\bar{\Upsilon}^i{}_m - {}_{\nu\mu}\bar{\Upsilon}^i{}_m + {}_\mu\Upsilon^i{}_n\,{}_\nu\Upsilon^n{}_m - {}_\nu\Upsilon^i{}_n\,{}_\mu\Upsilon^n{}_m}\,\eta^{\mu\sigma}\,\eta^{\nu\beta}\,{}_\beta\Upsilon^n{}_i$$

$$+4\underbrace{{}_{\mu\nu}\bar{\Upsilon}^n{}_j - {}_{\nu\mu}\bar{\Upsilon}^n{}_j + {}_\mu\Upsilon^n{}_m\,{}_\nu\Upsilon^m{}_j - {}_\nu\Upsilon^n{}_m\,{}_\mu\Upsilon^m{}_j}\,{}_\alpha\Upsilon^j{}_m\,\eta^{\mu\alpha}\,\eta^{\nu\sigma}$$

$${}^{x}\underbrace{\mathcal{L}^{\lambda n}\partial^\sigma_m}_{\Upsilon} = 4\underbrace{{}_{\mu\nu}\bar{\Upsilon}^n{}_m - {}_{\nu\mu}\bar{\Upsilon}^n{}_m + {}_\mu\Upsilon^n{}_k\,{}_\nu\Upsilon^k{}_m - {}_\nu\Upsilon^n{}_k\,{}_\mu\Upsilon^k{}_m}\,\eta^{\mu\lambda}\,\eta^{\nu\sigma}$$

$${}^{x}\mathcal{L}_{\Upsilon} = {}^{x}\mathcal{L}_{{}^x\Upsilon:{}^x\underline{\Upsilon}} = \overbrace{{}^{x}_{\mu}\underline{{}_\nu\Upsilon^i{}_j} - {}^{x}_{\nu}\underline{{}_\mu\Upsilon^i{}_j} + {}^x_\mu\Upsilon^i{}_k\,{}^x_\nu\Upsilon^k{}_j - {}^x_\nu\Upsilon^i{}_k\,{}^x_\mu\Upsilon^k{}_j}\,\eta^{\mu\alpha}\,\eta^{\nu\beta}\,\overbrace{{}^x_\alpha\underline{{}_\beta\Upsilon^j{}_i} - {}^x_\beta\underline{{}_\alpha\Upsilon^j{}_i} + {}^x_\alpha\Upsilon^j{}_\ell\,{}^x_\beta\Upsilon^\ell{}_i - {}^x_\beta\Upsilon^j{}_\ell\,{}^x_\alpha\Upsilon^\ell{}_i}$$

$$= 2\overbrace{{}^{x}_{\mu}\underline{{}_\nu\Upsilon^i{}_j} - {}^{x}_{\nu}\underline{{}_\mu\Upsilon^i{}_j} + {}^x_\mu\Upsilon^i{}_k\,{}^x_\nu\Upsilon^k{}_j - {}^x_\nu\Upsilon^i{}_k\,{}^x_\mu\Upsilon^k{}_j}\,\eta^{\mu\alpha}\,\eta^{\nu\beta}\,\overbrace{{}^x_\alpha\underline{{}_\beta\Upsilon^j{}_i} + {}^x_\alpha\Upsilon^j{}_\ell\,{}^x_\beta\Upsilon^\ell{}_i}$$

$${}^{x}\underbrace{\mathcal{L}\partial^{\sigma\ell}_i}_{\Upsilon} = {}^{x}\underbrace{\mathcal{L}\partial^{\sigma\ell}_i}_{{}^x\Upsilon:{}^x\underline{\Upsilon}}$$

$${}^{x}\underbrace{\mathcal{L}^\mu\partial^{\sigma\ell}_i}_{\Upsilon} = {}^{x}\underbrace{\mathcal{L}^\mu\partial^{\sigma\ell}_i}_{{}^x\Upsilon:{}^x\underline{\Upsilon}}$$

$$\underbrace{\mathcal{L}\partial^{\sigma\ell}_i}_{\Upsilon} = \partial_\mu\,\underbrace{\mathcal{L}^\mu\partial^{\sigma\ell}_i}_{\Upsilon} = {}_{\mu}\underline{\underbrace{\mathcal{L}^\mu\partial^{\sigma\ell}_i}_{\Upsilon}}$$

$$\eta^{\nu\beta}\,{}^{x}\underline{{}_{\beta}\underline{{}_{\mu}\dot\Upsilon_{\nu}.}-\underline{{}_{\nu}\dot\Upsilon_{\mu}.}+{}_{\mu}\dot\Upsilon.{}_{\nu}\dot\Upsilon.-{}_{\nu}\dot\Upsilon.{}_{\mu}\dot\Upsilon.}+\eta^{\nu\beta}\,{}^{x}_{\beta}\dot\Upsilon.\,\overbrace{{}^{x}\underline{{}_{\mu}\dot\Upsilon_{\nu}.}-{}^{x}\underline{{}_{\nu}\dot\Upsilon_{\mu}.}+{}^{x}_{\mu}\dot\Upsilon.{}^{x}_{\nu}\dot\Upsilon.-{}^{x}_{\nu}\dot\Upsilon.{}^{x}_{\mu}\dot\Upsilon.}-\overbrace{{}^{x}\underline{{}_{\mu}\dot\Upsilon_{\nu}.}-{}^{x}\underline{{}_{\nu}\dot\Upsilon_{\mu}.}+{}^{x}_{\mu}\dot\Upsilon.{}^{x}_{\nu}\dot\Upsilon.-{}^{x}_{\nu}\dot\Upsilon.{}^{x}_{\mu}\dot\Upsilon.}\,\eta^{\nu\beta}\,{}^{x}_{\beta}\dot\Upsilon.$$

$$=\eta^{\nu\beta}\,{}^{x}\underline{{}_{\beta}\underline{{}_{\mu}\dot\Upsilon_{\nu}.}-\underline{{}_{\nu}\dot\Upsilon_{\mu}.}+{}_{\mu}\dot\Upsilon.{}_{\nu}\dot\Upsilon.-{}_{\nu}\dot\Upsilon.{}_{\mu}\dot\Upsilon.}+\eta^{\nu\beta}\,{}^{x}_{\beta}\dot\Upsilon.\times\overbrace{{}^{x}\underline{{}_{\mu}\dot\Upsilon_{\nu}.}-{}^{x}\underline{{}_{\nu}\dot\Upsilon_{\mu}.}+{}^{x}_{\mu}\dot\Upsilon.{}^{x}_{\nu}\dot\Upsilon.-{}^{x}_{\nu}\dot\Upsilon.{}^{x}_{\mu}\dot\Upsilon.}$$

$$=\eta^{\nu\beta}\,\underbrace{{}_{\beta}\partial+{}^{x}_{\beta}\dot\Upsilon.\times}\overbrace{{}^{x}\underline{{}_{\mu}\dot\Upsilon_{\nu}.}-{}^{x}\underline{{}_{\nu}\dot\Upsilon_{\mu}.}+{}^{x}_{\mu}\dot\Upsilon.{}^{x}_{\nu}\dot\Upsilon.-{}^{x}_{\nu}\dot\Upsilon.{}^{x}_{\mu}\dot\Upsilon.}=0\bigwedge_{\mu}$$

$$\frac{1}{4}\,{}^{x}\mathcal{L}_{\Upsilon}\,\Upsilon^{\bullet}=\frac{1}{2}\overbrace{{}^{x}\underline{{}_{\mu}\Upsilon^{i}_{\nu\,j}}-{}^{x}\underline{{}_{\nu}\Upsilon^{i}_{\mu\,j}}+{}^{x}_{\mu}\Upsilon^{i}_{\ell}\,{}^{x}_{\nu}\Upsilon^{\ell}_{j}-{}^{x}_{\nu}\Upsilon^{i}_{\ell}\,{}^{x}_{\mu}\Upsilon^{\ell}_{j}}$$

$$\eta^{\mu\alpha}\,\eta^{\nu\beta}\overbrace{{}^{x}\underline{{}_{\alpha}\Upsilon^{\bullet j}_{\beta\,i}}-{}^{x}\underline{{}_{\beta}\Upsilon^{\bullet j}_{\alpha\,i}}+{}^{x}_{\alpha}\Upsilon^{\bullet j}_{k}\,{}^{x}_{\beta}\Upsilon^{k}_{i}+{}^{x}_{\alpha}\Upsilon^{j}_{k}\,{}^{x}_{\beta}\Upsilon^{\bullet k}_{i}-{}^{x}_{\beta}\Upsilon^{\bullet j}_{k}\,{}^{x}_{\alpha}\Upsilon^{k}_{i}-{}^{x}_{\beta}\Upsilon^{j}_{k}\,{}^{x}_{\alpha}\Upsilon^{\bullet k}_{i}}_{\alpha:\mu\,\natural\,\beta:\nu}=$$

$$\overbrace{{}^{x}\underline{{}_{\mu}\Upsilon^{i}_{\nu\,j}}-{}^{x}\underline{{}_{\nu}\Upsilon^{i}_{\mu\,j}}+{}^{x}_{\mu}\Upsilon^{i}_{\ell}\,{}^{x}_{\nu}\Upsilon^{\ell}_{j}-{}^{x}_{\nu}\Upsilon^{i}_{\ell}\,{}^{x}_{\mu}\Upsilon^{\ell}_{j}}\,\eta^{\mu\alpha}\,\eta^{\nu\beta}\overbrace{-{}^{x}\underline{{}_{\beta}\Upsilon^{\bullet j}_{\alpha\,i}}+{}^{x}_{\alpha}\Upsilon^{\bullet j}_{k}\,{}^{x}_{\beta}\Upsilon^{k}_{i}-{}^{x}_{\beta}\Upsilon^{j}_{k}\,{}^{x}_{\alpha}\Upsilon^{\bullet k}_{i}}\diamond\eta^{\nu\beta}\,{}^{x}\underline{{}_{\beta}\underline{{}_{\mu}\Upsilon^{i}_{\nu\,j}}-\underline{{}_{\nu}\Upsilon^{i}_{\mu\,j}}+{}_{\mu}\Upsilon^{i}_{\ell}\,{}_{\nu}\Upsilon^{\ell}_{j}-{}_{\nu}\Upsilon^{i}_{\ell}\,{}_{\mu}\Upsilon^{\ell}_{j}}\,\eta^{\mu\alpha}\,{}^{x}_{\alpha}\Upsilon^{\bullet j}\cdots$$

$$\underset{i\,\natural\,j}{+}\overbrace{{}^{x}\underline{{}_{\mu}\Upsilon^{k}_{\nu\,j}}-{}^{x}\underline{{}_{\nu}\Upsilon^{k}_{\mu\,j}}+{}^{x}_{\mu}\Upsilon^{k}_{\ell}\,{}^{x}_{\nu}\Upsilon^{\ell}_{j}-{}^{x}_{\nu}\Upsilon^{k}_{\ell}\,{}^{x}_{\mu}\Upsilon^{\ell}_{j}}\,\eta^{\mu\alpha}\,{}^{x}_{\alpha}\Upsilon^{\bullet j}_{i}\,\eta^{\nu\beta}\,{}^{x}_{\beta}\Upsilon^{i}_{k}\underset{j\,\natural\,j}{-}\overbrace{{}^{x}\underline{{}_{\mu}\Upsilon^{i}_{\nu\,k}}-{}^{x}\underline{{}_{\nu}\Upsilon^{i}_{\mu\,k}}+{}^{x}_{\mu}\Upsilon^{i}_{\ell}\,{}^{x}_{\nu}\Upsilon^{\ell}_{k}-{}^{x}_{\nu}\Upsilon^{i}_{\ell}\,{}^{x}_{\mu}\Upsilon^{\ell}_{k}}\,\eta^{\nu\beta}\,{}^{x}_{\beta}\Upsilon^{k}_{j}\,\eta^{\mu\alpha}\,{}^{x}_{\alpha}\Upsilon^{\bullet j}_{i}=$$

$$\eta^{\nu\beta}\,{}^{x}\underline{{}_{\beta}\underline{{}_{\mu}\Upsilon^{i}_{\nu\,j}}-\underline{{}_{\nu}\Upsilon^{i}_{\mu\,j}}+{}_{\mu}\Upsilon^{i}_{\ell}\,{}_{\nu}\Upsilon^{\ell}_{j}-{}_{\nu}\Upsilon^{i}_{\ell}\,{}_{\mu}\Upsilon^{\ell}_{j}}+\eta^{\nu\beta}\,{}^{x}_{\beta}\Upsilon^{i}_{k}\underbrace{{}^{x}\underline{{}_{\mu}\Upsilon^{k}_{\nu\,j}}-{}^{x}\underline{{}_{\nu}\Upsilon^{k}_{\mu\,j}}+{}^{x}_{\mu}\Upsilon^{k}_{\ell}\,{}^{x}_{\nu}\Upsilon^{\ell}_{j}-{}^{x}_{\nu}\Upsilon^{k}_{\ell}\,{}^{x}_{\mu}\Upsilon^{\ell}_{j}}-\underbrace{{}^{x}\underline{{}_{\mu}\Upsilon^{i}_{\nu\,k}}-{}^{x}\underline{{}_{\nu}\Upsilon^{i}_{\mu\,k}}+{}^{x}_{\mu}\Upsilon^{i}_{\ell}\,{}^{x}_{\nu}\Upsilon^{\ell}_{k}-{}^{x}_{\nu}\Upsilon^{i}_{\ell}\,{}^{x}_{\mu}\Upsilon^{\ell}_{k}}\,\eta^{\nu\beta}\,{}^{x}_{\beta}\Upsilon^{k}_{j}\,\eta^{\mu\alpha}\,{}^{x}_{\alpha}\Upsilon^{j}\cdots$$

$$\underbrace{{}^{x}\mathcal{L}}_{\Upsilon}\,\Upsilon=\underbrace{{}^{x}\mathcal{L}\partial^{\sigma\ell}_{i}}_{{}^{x}\Upsilon:{}^{x}\underline{\Upsilon}}\,{}^{x}\Upsilon^{\alpha}+\underbrace{{}^{x}\mathcal{L}^{\mu}\partial^{\sigma\ell}_{i}}_{{}^{x}\Upsilon:{}^{x}\underline{\Upsilon}}\,{}^{x}_{\mu}\underline{\Upsilon}^{\alpha}=\underbrace{{}^{x}\mathcal{L}\partial^{\sigma\ell}_{i}}_{\Upsilon}\,{}^{x}\Upsilon^{\alpha}+\underbrace{{}^{x}\mathcal{L}^{\mu}\partial^{\sigma\ell}_{i}}_{\Upsilon}\,{}^{x}_{\mu}\underline{\Upsilon}^{\alpha}$$

$$\underline{\int^{dx}{}^{x}\mathcal{L}}_{\Upsilon}\,\Upsilon=\int^{dx}\underline{{}^{x}\mathcal{L}}_{\Upsilon}\,\Upsilon=\int^{dx}\overbrace{{}^{x}\underbrace{\mathcal{L}\partial^{\sigma\ell}_{i}}_{\Upsilon}\,{}^{x}\Upsilon^{\alpha}+{}^{x}\underbrace{\mathcal{L}^{\mu}\partial^{\sigma\ell}_{i}}_{\Upsilon}\,{}^{x}_{\mu}\underline{\Upsilon}^{\alpha}}=\int^{dx}\overbrace{{}^{x}\underbrace{\mathcal{L}\partial^{\sigma\ell}_{i}}_{\Upsilon}\,{}^{x}\Upsilon^{\alpha}-{}^{x}\underline{{}_{\mu}\underbrace{\mathcal{L}^{\mu}\partial^{\sigma\ell}_{i}}_{\Upsilon}}\,{}^{x}\Upsilon^{\alpha}}=\int^{dx}\overbrace{{}^{x}\underbrace{\mathcal{L}\partial^{\sigma\ell}_{i}}_{\Upsilon}-{}^{x}\underline{{}_{\mu}\underbrace{\mathcal{L}^{\mu}\partial^{\sigma\ell}_{i}}_{\Upsilon}}}^{=0}\,{}^{x}\Upsilon^{\alpha}$$

$$\partial_\mu \underbrace{{}_\nu\delta^\mu \mathcal{L}_{\curlyvee} - \underbrace{\partial_\nu \overset{\alpha}{\curlyvee}}\,\underbrace{\mathcal{L}^\mu \partial_i^{\sigma\ell}}_{\curlyvee}} = \underbrace{\partial_\nu \mathcal{L}}_{\curlyvee}$$

$$\partial_\nu \mathcal{L}_{\curlyvee} = \underbrace{\partial_\nu \mathcal{L}}_{\curlyvee} + \underbrace{\mathcal{L}\partial_i^{\sigma\ell}}_{\curlyvee} {}_\nu\overset{\alpha}{\underline{\curlyvee}} + \underbrace{\mathcal{L}^\mu \partial_i^{\sigma\ell}}_{\curlyvee} {}_{\mu\nu}\overset{\alpha}{\underline{\underline{\curlyvee}}}$$

$$\text{LHS} = \partial_\nu \mathcal{L}_{\curlyvee} - \underbrace{\partial_\mu \underbrace{\partial_\nu \overset{\alpha}{\curlyvee}}}\,\underbrace{\mathcal{L}^\mu \partial_i^{\sigma\ell}}_{\curlyvee} - \underbrace{\partial_\nu \overset{\alpha}{\curlyvee}}\,\underbrace{\partial_\mu \underbrace{\mathcal{L}^\mu \partial_i^{\sigma\ell}}_{\curlyvee}}$$

$$= \underbrace{\partial_\nu \mathcal{L}}_{\curlyvee} + \underbrace{\mathcal{L}\partial_i^{\sigma\ell}}_{\curlyvee} {}_\nu\overset{\alpha}{\underline{\curlyvee}} + \underbrace{\mathcal{L}^\mu \partial_i^{\sigma\ell}}_{\curlyvee} {}_{\mu\nu}\overset{\alpha}{\underline{\underline{\curlyvee}}} - \underbrace{\partial_\mu \underbrace{\partial_\nu \overset{\alpha}{\curlyvee}}}\,\underbrace{\mathcal{L}^\mu \partial_i^{\sigma\ell}}_{\curlyvee} - \underbrace{\partial_\nu \overset{\alpha}{\curlyvee}}\,\overbrace{\partial_\mu \underbrace{\mathcal{L}^\mu \partial_i^{\sigma\ell}}_{\curlyvee}}^{=\underbrace{\mathcal{L}\partial_i^{\sigma\ell}}_{\curlyvee}} = \underbrace{\partial_\nu \mathcal{L}}_{\curlyvee}$$

$${}_{\mu\nu}\overset{i}{F}{}_j = {}_{\mu\nu}\overset{i}{\curlyvee}{}_j - {}_{\nu\mu}\overset{i}{\curlyvee}{}_j + {}_\mu\overset{i}{\curlyvee}{}_k\, {}_\nu\overset{k}{\curlyvee}{}_j - {}_\nu\overset{i}{\curlyvee}{}_k\, {}_\mu\overset{k}{\curlyvee}{}_j$$

$${}^x\mathcal{L}_{\curlyvee:\curlyvee} = tr\, {}_{\alpha\beta}\dot{F}.\, \eta^{\alpha\mu}\, \eta^{\beta\nu}\, {}_{\mu\nu}\dot{F}.^{T} = {}_{\alpha\beta}\overset{i}{F}{}_j\, \eta^{\alpha\mu}\, \eta^{\beta\nu}\, {}_{\mu\nu}\overset{i}{F}{}_j$$

$$= tr\,\overbrace{{}_{\alpha\beta}\dot{\curlyvee}. - {}_{\beta\alpha}\dot{\curlyvee}. + {}_\alpha\dot{\curlyvee}.\,{}_\beta\dot{\curlyvee}. - {}_\beta\dot{\curlyvee}.\,{}_\alpha\dot{\curlyvee}.}\, \eta^{\alpha\mu}\, \eta^{\beta\nu}\, \overbrace{{}_{\mu\nu}\dot{\curlyvee}. - {}_{\nu\mu}\dot{\curlyvee}. + {}_\mu\dot{\curlyvee}.\,{}_\nu\dot{\curlyvee}. - {}_\nu\dot{\curlyvee}.\,{}_\mu\dot{\curlyvee}.}^{T}$$

$$= \underbrace{{}_{\alpha\beta}\overset{j}{\curlyvee}{}_i - {}_{\beta\alpha}\overset{j}{\curlyvee}{}_i + {}_\alpha\overset{j}{\curlyvee}{}_\ell\, {}_\beta\overset{\ell}{\curlyvee}{}_i - {}_\beta\overset{j}{\curlyvee}{}_\ell\, {}_\alpha\overset{\ell}{\curlyvee}{}_i}\, \eta^{\alpha\mu}\, \eta^{\beta\nu}\, \underbrace{{}_{\mu\nu}\overset{i}{\curlyvee}{}_j - {}_{\nu\mu}\overset{i}{\curlyvee}{}_j + {}_\mu\overset{i}{\curlyvee}{}_k\, {}_\nu\overset{k}{\curlyvee}{}_j - {}_\nu\overset{i}{\curlyvee}{}_k\, {}_\mu\overset{k}{\curlyvee}{}_j}$$

$$\text{fields } \mathbb{R}^d \xrightarrow{{}_\sigma\overset{i}{\curlyvee}{}_\ell} {}_d^N\mathbb{R}_N \ni {}^x_\sigma\overset{i}{\curlyvee}{}_\ell$$

$${}^x\overbrace{{}_\mu\partial\, {}_\nu\curlyvee}^{i}{}_j = {}^x_{\mu}\underline{{}_\nu\overset{i}{\curlyvee}{}_j} + {}^x_\mu\overset{i}{\curlyvee}{}_k\, {}^x_\nu\overset{k}{\curlyvee}{}_j$$

$${}^x\overbrace{{}_\sigma\partial\, \underbrace{{}_\mu\partial\, {}_\nu\dot{\curlyvee}. - {}_\nu\partial\, {}_\mu\dot{\curlyvee}.}}^{i}{}_j = {}^x_\sigma\underline{{}_\mu\partial\, {}_\nu\dot{\curlyvee}. - {}_\nu\partial\, {}_\mu\dot{\curlyvee}.}^{\,i}{}_j + \overbrace{{}^x_\sigma\dot{\curlyvee}. \times \underbrace{{}_\mu\partial\, {}_\nu\dot{\curlyvee}. - {}_\nu\partial\, {}_\mu\dot{\curlyvee}.}}^{i}{}_j$$

$${}^x\mathcal{L}_{\curlyvee} = {}^x\mathcal{L}_{{}^x\curlyvee:{}^x\underline{\curlyvee}} = \overbrace{{}^x_\alpha\underline{{}_\beta\overset{j}{\curlyvee}{}_i} - {}^x_\beta\underline{{}_\alpha\overset{j}{\curlyvee}{}_i} + {}^x_\alpha\overset{j}{\curlyvee}{}_\ell\, {}^x_\beta\overset{\ell}{\curlyvee}{}_i - {}^x_\beta\overset{j}{\curlyvee}{}_\ell\, {}^x_\alpha\overset{\ell}{\curlyvee}{}_i}\, \eta^{\alpha\mu}\, \eta^{\beta\nu}\, \overbrace{{}^x_\mu\underline{{}_\nu\overset{i}{\curlyvee}{}_j} - {}^x_\nu\underline{{}_\mu\overset{i}{\curlyvee}{}_j} + {}^x_\mu\overset{i}{\curlyvee}{}_k\, {}^x_\nu\overset{k}{\curlyvee}{}_j - {}^x_\nu\overset{i}{\curlyvee}{}_k\, {}^x_\mu\overset{k}{\curlyvee}{}_j}$$

$$\text{motion } 0 = \eta^{\mu\sigma} \; \overbrace{{}^{x}\underline{{}_{\sigma}\left({}_{\mu}\underline{{}_{\nu}\dot{\curlyvee}.} - {}_{\nu}\underline{{}_{\mu}\dot{\curlyvee}.} + {}_{\mu}\dot{\curlyvee}.\,{}_{\nu}\dot{\curlyvee}. - {}_{\nu}\dot{\curlyvee}.\,{}_{\mu}\dot{\curlyvee}.\right)} + {}^{x}_{\sigma}\dot{\curlyvee}. \times \underbrace{{}_{\mu}\underline{{}^{x}_{\nu}\dot{\curlyvee}.} - {}_{\nu}\underline{{}^{x}_{\mu}\dot{\curlyvee}.} + {}^{x}_{\mu}\dot{\curlyvee}.\,{}^{x}_{\nu}\dot{\curlyvee}. - {}^{x}_{\nu}\dot{\curlyvee}.\,{}^{x}_{\mu}\dot{\curlyvee}.}}^{n}{}_{m}$$

$$= \eta^{\mu\sigma} \; \overbrace{\underbrace{{}_{\sigma}\partial + {}^{x}_{\sigma}\dot{\curlyvee}. \times}\, \underbrace{{}_{\mu}\underline{{}^{x}_{\nu}\dot{\curlyvee}.} - {}_{\nu}\underline{{}^{x}_{\mu}\dot{\curlyvee}.} + {}^{x}_{\mu}\dot{\curlyvee}.\,{}^{x}_{\nu}\dot{\curlyvee}. - {}^{x}_{\nu}\dot{\curlyvee}.\,{}^{x}_{\mu}\dot{\curlyvee}.}}^{n}{}_{m}$$

$$\eta^{\tau\nu}\,\eta^{\mu\sigma} \; {}^{x}\underline{{}_{\sigma}\left({}_{\mu}\underline{{}_{\nu}\dot{\curlyvee}.} - {}_{\nu}\underline{{}_{\mu}\dot{\curlyvee}.} + {}_{\mu}\dot{\curlyvee}.\,{}_{\nu}\dot{\curlyvee}. - {}_{\nu}\dot{\curlyvee}.\,{}_{\mu}\dot{\curlyvee}.\right)} = \frac{1}{4}\, {}^{x}\underline{{}_{\sigma}\mathcal{L}^{n}\partial^{\sigma\tau}_{m}}_{\curlyvee} = \frac{1}{4}\, {}^{x}\underbrace{\mathcal{L}^{n}\partial^{\tau}_{m}}_{\curlyvee} = \eta^{\tau\mu}\,\eta^{\sigma\nu} \; \overbrace{{}^{x}_{\sigma}\dot{\curlyvee}. \times \underbrace{{}_{\mu}\underline{{}^{x}_{\nu}\dot{\curlyvee}.} - {}_{\nu}\underline{{}^{x}_{\mu}\dot{\curlyvee}.} + {}^{x}_{\mu}\dot{\curlyvee}.\,{}^{x}_{\nu}\dot{\curlyvee}. - {}^{x}_{\nu}\dot{\curlyvee}.\,{}^{x}_{\mu}\dot{\curlyvee}.}}^{n}$$

$$= \eta^{\tau\nu}\,\eta^{\sigma\mu} \; \overbrace{{}^{x}_{\sigma}\dot{\curlyvee}. \times \underbrace{{}_{\nu}\underline{{}^{x}_{\mu}\dot{\curlyvee}.} - {}_{\mu}\underline{{}^{x}_{\nu}\dot{\curlyvee}.} + {}^{x}_{\nu}\dot{\curlyvee}.\,{}^{x}_{\mu}\dot{\curlyvee}. - {}^{x}_{\mu}\dot{\curlyvee}.\,{}^{x}_{\nu}\dot{\curlyvee}.}}^{n}{}_{m}$$

$$= -\,\eta^{\tau\nu}\,\eta^{\sigma\mu} \; \overbrace{{}^{x}_{\sigma}\dot{\curlyvee}. \times \underbrace{{}_{\mu}\underline{{}^{x}_{\nu}\dot{\curlyvee}.} - {}_{\nu}\underline{{}^{x}_{\mu}\dot{\curlyvee}.} + {}^{x}_{\mu}\dot{\curlyvee}.\,{}^{x}_{\nu}\dot{\curlyvee}. - {}^{x}_{\nu}\dot{\curlyvee}.\,{}^{x}_{\mu}\dot{\curlyvee}.}}^{n}{}_{m}$$

$$0 = \eta^{\tau\nu}\,\eta^{\mu\sigma} \; \overbrace{{}^{x}\underline{{}_{\sigma}\left({}_{\mu}\underline{{}_{\nu}\dot{\curlyvee}.} - {}_{\nu}\underline{{}_{\mu}\dot{\curlyvee}.} + {}_{\mu}\dot{\curlyvee}.\,{}_{\nu}\dot{\curlyvee}. - {}_{\nu}\dot{\curlyvee}.\,{}_{\mu}\dot{\curlyvee}.\right)} + {}^{x}_{\sigma}\dot{\curlyvee}. \times \underbrace{{}_{\mu}\underline{{}^{x}_{\nu}\dot{\curlyvee}.} - {}_{\nu}\underline{{}^{x}_{\mu}\dot{\curlyvee}.} + {}^{x}_{\mu}\dot{\curlyvee}.\,{}^{x}_{\nu}\dot{\curlyvee}. - {}^{x}_{\nu}\dot{\curlyvee}.\,{}^{x}_{\mu}\dot{\curlyvee}.}}^{n}{}_{m}$$

$$\underset{\tau \text{ bel}}{\Longrightarrow}\; 0 = \eta^{\mu\sigma} \; \overbrace{{}^{x}\underline{{}_{\sigma}\left({}_{\mu}\underline{{}_{\nu}\dot{\curlyvee}.} - {}_{\nu}\underline{{}_{\mu}\dot{\curlyvee}.} + {}_{\mu}\dot{\curlyvee}.\,{}_{\nu}\dot{\curlyvee}. - {}_{\nu}\dot{\curlyvee}.\,{}_{\mu}\dot{\curlyvee}.\right)} + {}^{x}_{\sigma}\dot{\curlyvee}. \times \underbrace{{}_{\mu}\underline{{}^{x}_{\nu}\dot{\curlyvee}.} - {}_{\nu}\underline{{}^{x}_{\mu}\dot{\curlyvee}.} + {}^{x}_{\mu}\dot{\curlyvee}.\,{}^{x}_{\nu}\dot{\curlyvee}. - {}^{x}_{\nu}\dot{\curlyvee}.\,{}^{x}_{\mu}\dot{\curlyvee}.}}^{n}{}_{m}$$

$$\underbrace{{}^{x}\mathcal{L}}_{\curlyvee}\curlyvee\!\!\dashv = {}^{x}\underbrace{\mathcal{L}\partial^{\sigma\ell}_{i}}_{{}^{x}\curlyvee:{}^{x}\underline{\curlyvee}}\;{}^{x\alpha}\curlyvee\!\!\dashv + {}^{x}\underbrace{\mathcal{L}^{\mu}\partial^{\sigma\ell}_{i}}_{{}^{x}\curlyvee:{}^{x}\underline{\curlyvee}}\;{}^{x\alpha}_{\mu}\underline{\curlyvee\!\!\dashv} = {}^{x}\underbrace{\mathcal{L}\partial^{\sigma\ell}_{i}}_{\curlyvee}\;{}^{x\alpha}\curlyvee\!\!\dashv + {}^{x}\underbrace{\mathcal{L}^{\mu}\partial^{\sigma\ell}_{i}}_{\curlyvee}\;{}^{x\alpha}_{\mu}\underline{\curlyvee\!\!\dashv}$$

$$\underline{\int^{dx}{}^{x}\mathcal{L}}_{\curlyvee}\;\curlyvee\!\!\dashv = \int {}^{x}\underline{\mathcal{L}}_{\curlyvee}\curlyvee\!\!\dashv = \int^{dx} \overbrace{{}^{x}\underbrace{\mathcal{L}\partial^{\sigma\ell}_{i}}_{\curlyvee}\;{}^{x\alpha}\curlyvee\!\!\dashv + {}^{x}\underbrace{\mathcal{L}^{\mu}\partial^{\sigma\ell}_{i}}_{\curlyvee}\;{}^{x\alpha}_{\mu}\underline{\curlyvee\!\!\dashv}} = \int^{dx} \overbrace{{}^{x}\underbrace{\mathcal{L}\partial^{\sigma\ell}_{i}}_{\curlyvee}\;{}^{x\alpha}\curlyvee\!\!\dashv - {}_{\mu}\underline{{}^{x}\underbrace{\mathcal{L}^{\mu}\partial^{\sigma\ell}_{i}}_{\curlyvee}}\;{}^{x\alpha}\curlyvee\!\!\dashv} = \int^{dx} \overbrace{{}^{x}\underbrace{\mathcal{L}\partial^{\sigma\ell}_{i}}_{\curlyvee} - {}_{\mu}\underline{{}^{x}\underbrace{\mathcal{L}^{\mu}\partial^{\sigma\ell}_{i}}_{\curlyvee}}}^{=0}\;{}^{x\alpha}\curlyvee\!\!\dashv$$

$$\partial_\mu \underbrace{{}_\nu\delta^\mu \mathcal{L}_{\curlyvee} - \underbrace{\partial_\nu \curlyvee^\alpha}\underbrace{\mathcal{L}^\mu \partial_i^{\sigma\ell}}_{\curlyvee}} = \underbrace{\partial_\nu \mathcal{L}}_{\curlyvee}$$

$$\partial_\nu \mathcal{L}_{\curlyvee} = \underbrace{\partial_\nu \mathcal{L}}_{\curlyvee} + \underbrace{\mathcal{L}\partial_i^{\sigma\ell}}_{\curlyvee}\, {}_\nu\underline{\curlyvee}^\alpha + \underbrace{\mathcal{L}^\mu \partial_i^{\sigma\ell}}_{\curlyvee}\, {}_{\mu\nu}\underline{\underline{\curlyvee}}^\alpha$$

$$\text{LHS} = \partial_\nu \mathcal{L}_{\curlyvee} - \underbrace{\partial_\mu \underbrace{\partial_\nu \curlyvee^\alpha}}\, \underbrace{\mathcal{L}^\mu \partial_i^{\sigma\ell}}_{\curlyvee} - \underbrace{\partial_\nu \curlyvee^\alpha}\, \underbrace{\partial_\mu \underbrace{\mathcal{L}^\mu \partial_i^{\sigma\ell}}_{\curlyvee}}$$

$$= \underbrace{\partial_\nu \mathcal{L}}_{\curlyvee} + \underbrace{\mathcal{L}\partial_i^{\sigma\ell}}_{\curlyvee}\, {}_\nu\underline{\curlyvee}^\alpha + \underbrace{\mathcal{L}^\mu \partial_i^{\sigma\ell}}_{\curlyvee}\, {}_{\mu\nu}\underline{\underline{\curlyvee}}^\alpha - \underbrace{\partial_\mu \underbrace{\partial_\nu \curlyvee^\alpha}}\, \underbrace{\mathcal{L}^\mu \partial_i^{\sigma\ell}}_{\curlyvee} - \underbrace{\partial_\nu \curlyvee^\alpha}\, \overbrace{\partial_\mu \underbrace{\mathcal{L}^\mu \partial_i^{\sigma\ell}}_{\curlyvee}}^{= \underbrace{\mathcal{L}\partial_i^{\sigma\ell}}_{\curlyvee}} = \underbrace{\partial_\nu \mathcal{L}}_{\curlyvee}$$