

$$\begin{aligned}
D &= \gamma^\mu{}_\mu \partial \\
\mathcal{D} &= \dot{\gamma}^\mu{}_\mu \partial \\
\varkappa = 1 \Rightarrow D &= \mathcal{D} \\
\Gamma D &= -D\Gamma \\
\Gamma \mathcal{D} &= -\mathcal{D}\Gamma \\
\Gamma \gamma^\mu &= -\gamma^\mu \Gamma \Rightarrow \Gamma \underbrace{\gamma^\mu{}_\mu \partial}_{\gamma^\mu} = -\underbrace{\gamma^\mu{}_\mu \partial}_{\gamma^\mu} \Gamma \\
\dot{\gamma}^\mu &= \pm \gamma^\mu \Rightarrow \Gamma \dot{\gamma}^\mu = -\dot{\gamma}^\mu \Gamma \Rightarrow \Gamma \underbrace{\dot{\gamma}^\mu{}_\mu \partial}_{\dot{\gamma}^\mu} = -\underbrace{\dot{\gamma}^\mu{}_\mu \partial}_{\dot{\gamma}^\mu} \Gamma
\end{aligned}$$

$$\begin{aligned}
\varkappa = i &\Rightarrow D \text{ skew-adj} \\
\varkappa = 1 &\Rightarrow \gamma^0 D \text{ skew-adj } \gamma^0 \mathcal{D}
\end{aligned}$$

$$\begin{aligned}
\varkappa = i: \widehat{\gamma^\mu{}_\mu \partial}^* &= {}_\mu \dot{\partial} \dot{\gamma}^\mu = -{}_\mu \partial \gamma^\mu = -\gamma^\mu{}_\mu \partial \Leftarrow \gamma^\mu \text{ cst} \\
\varkappa = 1: \widehat{\gamma^0 \gamma^k}^* &= \dot{\gamma}^k \dot{\gamma}^0 = -\gamma^k \gamma^0 = \gamma^0 \gamma^k \\
\widehat{\gamma^0 \gamma^0}^* &= \dot{\gamma}^0 \dot{\gamma}^0 = \gamma^0 \gamma^0 \Rightarrow \\
\widehat{\gamma^0 \gamma^\mu{}_\mu \partial}^* &= {}_\mu \dot{\partial} \widehat{\gamma^0 \gamma^\mu}^* = -{}_\mu \partial \gamma^0 \gamma^\mu = -\underbrace{\gamma^0 \gamma^\mu}_{\gamma^\mu} {}_\mu \partial \Leftarrow \gamma^0 \gamma^\mu \text{ cst} \\
\widehat{\gamma^0 \dot{\gamma}^\mu{}_\mu \partial}^* &= {}_\mu \dot{\partial} \widehat{\gamma^0 \dot{\gamma}^\mu}^* = -{}_\mu \partial \gamma^0 \dot{\gamma}^\mu = -\underbrace{\gamma^0 \dot{\gamma}^\mu}_{\dot{\gamma}^\mu} {}_\mu \partial \Leftarrow \gamma^0 \dot{\gamma}^\mu \text{ cst}
\end{aligned}$$

$$\begin{aligned}
D^* &= -D \\
\mathcal{D}^* &= -\mathcal{D} \\
\underbrace{\gamma^0 \bar{\gamma}^2} \mathcal{D} &= -D \underbrace{\gamma^0 \bar{\gamma}^0} \\
\sim \mathcal{D} &= -D^\sim \\
\sim \mathcal{D} = \sim \dot{\gamma}^\mu{}_\mu \partial &= -\gamma^\mu{}_\mu \sim \partial = -\gamma^\mu{}_\mu \partial^\sim = -D^\sim \\
\underbrace{\gamma^0 \bar{\gamma}^2} \mathcal{D} \underbrace{\gamma^0 \bar{\gamma}^2} &= D \\
\sim \mathcal{D}^\sim &= D \\
\sim D^\sim &= \mathcal{D} \\
\sim \mathcal{D}^\sim &= -D^{\sim\sim} = D \\
\sim D^\sim &= -\sim \mathcal{D} = \mathcal{D}
\end{aligned}$$