$$\begin{array}{ccc} \mathbb{K}^{\binom{n+m+1}{m}} & & \mathbb{K}^{2^n} \\ \downarrow {\scriptstyle \bullet \rfloor} & & \downarrow {\scriptstyle \bullet \rfloor} \\ \mathbb{K}_m \rfloor & & \mathbb{K}_{\mathbb{N}} \rfloor \end{array}$$

$$\mathbb{K}^{\binom{n+m+1}{m}} \ni \left({}_I\lfloor\right)_I \mapsto \sum_{|I| = m} \lfloor^I \; {}_I\rfloor \in \mathbb{K}_m \rfloor$$

$$\mathbb{K}^{2^n} \ni \lfloor^\bullet \curvearrowright \lfloor^\bullet \; {}_\bullet\rfloor \in \mathbb{K}_{\mathbb{N}} \rfloor$$

$$\mathbb{K}_m \rfloor \ni {}_I\rfloor = \bigotimes_{i \in I} {}_i\rfloor = \begin{bmatrix} {}_{i_1}\rfloor \\ \otimes \end{bmatrix} {}_{i_m}\rfloor \text{ Basis}$$

$$n \supset I = \left\{ i_1 \leqslant \cdots \leqslant i_m \right\}$$

$${}_I\rfloor \; {}_J\overset{*}{\rfloor} = \text{ per } \; {}_i\rfloor \; {}_j\overset{*}{\rfloor} = \text{ per } \; {}_i\delta^j = {}_I\delta^J = {}_I\rfloor \; \rfloor^J$$

$${}_J\overset{*}{\rfloor} = \rfloor^J$$

$${}_I\rfloor \ast {}_J\rfloor = \text{ per } \; {}_i\rfloor \ast {}_j\rfloor = \text{ per } \; {}_i\eta^j = {}_I\eta^J \text{ diag}$$

$$\begin{array}{ccc} \mathbb{K}^{\binom{n}{m}} & & \mathbb{K}^{2^n} \\ \downarrow {\scriptstyle \bullet \rfloor} & & \downarrow {\scriptstyle \bullet \rfloor} \\ \mathbb{K}_m \rfloor & & \mathbb{K}_{\mathbb{N}} \rfloor \end{array}$$

$$\mathbb{K}^{\binom{n}{m}} \ni \left({}_I\lfloor\right)_I \mapsto \sum_{|I| = m} \lfloor^I \; {}_I\rfloor \in \mathbb{K}_m \rfloor$$

$$\mathbb{K}^{2^n} \ni \lfloor^\bullet \lightning \lfloor^\bullet \; {}_\bullet\rfloor \in \mathbb{K}_{\mathbb{N}} \rfloor$$

$$\mathbb{K}_m \rfloor \ni {}_I\rfloor = \bigotimes_{i \in I} {}_i\rfloor = \begin{bmatrix} {}_{i_1}\rfloor \\ \otimes \end{bmatrix} {}_{i_m}\rfloor \text{ Basis}$$

$$n \supset I = \left\{ i_1 < \cdots < i_m \right\}$$

$$
{}_{I}\mathtt{l}\ {}_{J}\overset{*}{\mathtt{l}} = \det\ {}_{i}\mathtt{l}\ {}_{j}\overset{*}{\mathtt{l}} = \det {}_{i}\delta^{j} = {}_{I}\delta^{J} = {}_{I}\mathtt{l}\ \mathtt{l}^{J}
$$

$$
{}_{J}\overset{*}{\mathtt{l}} = \mathtt{l}^{J}
$$

$$
{}_{I}\mathtt{l}\ \text{⧗}\ {}_{J}\mathtt{l} = \det\ {}_{i}\mathtt{l}\ \text{⧗}\ {}_{j}\mathtt{l} = \det {}_{i}\eta^{j} = {}_{I}\eta^{J}\ \mathrm{diag}
$$