

$$\mathbb{K}^{\binom{n+m+1}{m}} \quad \mathbb{K}^{2^n}$$

$$\begin{array}{ccc} \downarrow & & \downarrow \\ \bullet \mathcal{L} & & \bullet \mathcal{L} \\ \downarrow & & \downarrow \\ \mathbb{K}_m^{\mathcal{N}\mathcal{L}} & & \mathbb{K}_{\mathbb{N}}^{\mathcal{N}\mathcal{L}} \end{array}$$

$$\mathbb{K}^{\binom{n+m+1}{m}} \ni (\underline{L})_I \mapsto \sum_{|I|=m} L^I \cdot \underline{L} \in \mathbb{K}_m^{\mathcal{N}\mathcal{L}}$$

$$\mathbb{K}^{2^n} \ni L^\bullet \curvearrowright \underline{L}^\bullet \cdot \underline{L} \in \mathbb{K}_{\mathbb{N}}^{\mathcal{N}\mathcal{L}}$$

$$\mathbb{K}_m^{\mathcal{N}\mathcal{L}} \ni \underline{L} = \begin{bmatrix} \underline{L} \\ \mathbf{x} \end{bmatrix}_I = \begin{bmatrix} i_1 \underline{L} \\ \mathbf{x} \end{bmatrix}_{i_m} \underline{L} \text{ Basis}$$

$$n \supset I = \{i_1 \leq \dots \leq i_m\}$$

$$\begin{aligned} {}_I \underline{L} \cdot {}_J \underline{L}^* &= \text{per } {}_I \underline{L} \cdot {}_J \underline{L}^* = \text{per } {}_i \delta^j = {}_I \delta^J = {}_I \underline{L} \cdot \underline{L}^J \\ {}_J \underline{L}^* &= \underline{L}^J \end{aligned}$$

$${}_I \underline{L} \star {}_J \underline{L} = \text{per } {}_I \underline{L} \star {}_J \underline{L} = \text{per } {}_i \eta^j = {}_I \eta^J \text{ diag}$$

$$\mathbb{K}_m^{\binom{n}{m}} \quad \mathbb{K}^{2^n}$$

$$\begin{array}{ccc} \downarrow & & \downarrow \\ \bullet \mathcal{L} & & \bullet \mathcal{L} \\ \downarrow & & \downarrow \\ \mathbb{K}_m^{\mathcal{N}\mathcal{L}} & & \mathbb{K}_{\mathbb{N}}^{\mathcal{N}\mathcal{L}} \end{array}$$

$$\mathbb{K}^{\binom{n}{m}} \ni (\underline{L})_I \mapsto \sum_{|I|=m} L^I \cdot \underline{L} \in \mathbb{K}_m^{\mathcal{N}\mathcal{L}}$$

$$\mathbb{K}^{2^n} \ni L^\bullet \ntriangleright \underline{L}^\bullet \cdot \underline{L} \in \mathbb{K}_{\mathbb{N}}^{\mathcal{N}\mathcal{L}}$$

$$\mathbb{K}_m^{\mathcal{N}\mathcal{L}} \ni \underline{L} = \begin{bmatrix} \underline{L} \\ \mathbf{x} \end{bmatrix}_I = \begin{bmatrix} i_1 \underline{L} \\ \mathbf{x} \end{bmatrix}_{i_m} \underline{L} \text{ Basis}$$

$$n \supset I = \{i_1 < \dots < i_m\}$$

$${}_I\mathsf{L} \underset{_J}{\star} {}_J^*\mathsf{L} = \det {}_I\mathsf{L} \underset{_J}{\star} {}_J^*\mathsf{L} = \det {}_i\delta^j = {}_I\delta^J = {}_I\mathsf{L} \underset{}{\star} {}^J$$

$${}_J^*\mathsf{L} = \mathsf{L}^J$$

$${}_I\mathsf{L} \underset{}{\star} {}_J\mathsf{L} = \det {}_I\mathsf{L} \underset{}{\star} {}_J\mathsf{L} = \det {}_i\eta^j = {}_I\eta^J \text{ diag}$$