

$$\begin{array}{c} \mathbb{L} \rtimes \mathbb{J} = \mathop{\mathbb{X}}\limits_{i \in I} \mathbb{L} \rtimes \mathop{\mathbb{X}}\limits_{j \in J} \mathbb{J} = \text{per } \mathbb{L} \rtimes \mathbb{J} = \text{per } \mathbb{L} \eta \left(\mathbb{J}^* \right) \\ \mathbb{K} \nabla \mathbb{L} \rightarrow \mathbb{K} \nabla \mathbb{L} \begin{array}{c} \mathbb{K} \\ \triangleleft \end{array} \end{array}$$

$$\mathop{\mathbb{X}}\limits_{i \in I} \mathbb{L} \mid \mathop{\mathbb{X}}\limits_{j \in J} \mathbb{T}^j = \text{per } \mathbb{L} \mathbb{T}^j$$

$$\text{LHS} \; = m! \mathop{\mathbb{X}}\limits_{i \in I} \mathbb{L} \mid p_{-j} \mathop{\mathbb{X}}\limits_{j \in J} \mathbb{T}^j = \sum_{I \stackrel{\sim}{\rightarrow} \pi J} -1 \mathop{\mathbb{X}}\limits_{i \in I} \mathbb{L} \mathop{\mathbb{X}}\limits_{i \in I} \mathbb{T}^{\pi_i} = \sum_{\pi} \prod_{i \in I} \mathbb{L} \mathbb{T}^{\pi_i} = \text{RHS}$$

$$\mathbb{L} \nabla \mathbb{L} = \mathbb{L} \rtimes \mathbb{K} \nabla \mathbb{L}$$

$${}_N\mathbb{L} \in \mathbb{K} \nabla \mathbb{L}$$

$${}_N\mathbb{L} \mathbb{T} = {}_N\mathbb{L} \text{ perm } \mathbb{T}$$

$$\mathbb{K}^{\pm} \nabla \mathbb{L} = \sum_m \mathbb{K}^{\pm}_m \nabla \mathbb{L} \in \mathbb{K} \triangleleft \text{abel}$$

$$\begin{array}{ccc} \mathbb{K}^{\pm}_N \nabla \mathbb{L} & & \\ \uparrow & \searrow \tilde{\varphi} & \\ & \mathbb{V} & \\ \mathbb{L} & \nearrow \varphi & \mathbb{L} \in \mathbb{K} \triangleleft \text{Gel} \\ & \mathbb{\Lambda} & \end{array}$$

$$\mathbb{L} \varphi \rtimes \mathbb{L}' \varphi = 0$$

$$\mathbb{L} \iota = \mathbb{L} + \mathcal{J}$$

$$\mathbb{L} \iota \rtimes \mathbb{L}' \iota = 0$$

$$(\mathbb{L} \rtimes \cdots \rtimes_m \mathbb{L} + \mathcal{J}) \, \tilde{\varphi} \colon = \mathbb{L} \varphi \times \cdots \times_m \mathbb{L} \varphi$$

$$(\lfloor \mathbf{x}\cdots \mathbf{x}_m\rfloor + \mathcal{J})\, \mathbb{K}_{\mathbb{N}}\nabla \mathbb{L} := \lfloor \mathbb{L} \mathbb{L} \mathbf{x}\cdots \mathbf{x}_m\rfloor \mathbb{L} + \mathcal{J}$$

$$\mathbb{K}_{\mathbb{N}}^+\nabla\mathbb{L}:=\mathbb{K}_{\mathbb{N}}\nabla\mathbb{L}+\mathbb{K}_{\mathbb{N}}\nabla\mathbb{L}\left\{\lfloor\mathbf{x}\mathbb{L}-\mathbb{L}\mathbf{x}\rfloor\right\}\mathbb{K}_{\mathbb{N}}\nabla\mathbb{L}$$

$$\mathbb{K}_m^+\nabla\mathbb{L}=\mathbb{K}_m\nabla\mathbb{L}+\mathbb{K}\overline{\lfloor\mathbf{x}\cdots\mathbf{x}_{\pi_1}\rfloor-\lfloor\mathbf{x}\cdots\mathbf{x}_m\rfloor}$$

$$\mathbb{L} = \mathop{\mathbb{L}}\limits_{i \in I} \mathop{\mathbf{x}}\limits_I \mathbb{L} = \begin{bmatrix} \mathbb{L}_{i_1} \\ + \\ \mathbb{L}_{i_m} \end{bmatrix}$$

$$I=\left\{i_1\leqslant\cdots\leqslant i_m\right\}\text{ geordnet}$$

$$\mathbb{K}_{\mathbb{N}}^+\nabla\mathbb{L}\in\mathbb{K}_{\begin{smallmatrix}\mathbb{N}\\0\end{smallmatrix}}^{1:1}$$

$$\mathbb{L} \star \mathbb{J}' = \mathop{\mathbf{x}}\limits_I \mathbb{L} \star \mathop{\mathbf{x}}\limits_J \mathbb{J}' = \text{Per } \mathbb{L} \star \mathbb{J}' = \text{Per } \mathbb{L} \eta \left(\mathbb{J}'^* \right)$$

$$\mathbb{K}_m^+\nabla\mathbb{L}\rightarrow\mathbb{K}\nabla\mathbb{L}_{\begin{smallmatrix}\mathbb{N}\\m\end{smallmatrix}}^+\mathbb{K}$$

$$\mathbb{K}_{\mathbb{N}}^+\nabla\mathbb{L}_{\mathbb{K}\mathbb{N}}\leftarrow\mathbb{L}_{\mathbb{N}}^+\mathbb{K}$$

$$\mathop{\mathbf{x}}\limits_I \mathbb{L} \big|_{\mathop{\mathbf{x}}\limits_J \mathbb{T}^j} = \text{Per } \mathbb{L} \mathbb{T}^j$$

$$\text{LHS} \; = m! \mathop{\mathbf{x}}\limits_I \mathbb{L} \big|_{p_+ \mathop{\mathbf{x}}\limits_J \mathbb{T}^j} = \sum_{I \stackrel{\sim}{\rightarrow} \pi J} \mathop{\mathbf{x}}\limits_I \mathbb{L} \mathop{\mathbf{x}}\limits_I \mathbb{T}^{\pi_i} = \sum_{\pi} \prod_{i \in I} \mathbb{L} \mathbb{T}^{\pi_i} = \text{RHS}$$

$$\mathbb{L}_{\mathbb{N}}^+\nabla\mathbb{L}=\mathbb{L}\mathbf{x}\mathbb{K}_{\mathbb{N}}^+\nabla\mathbb{L}$$

$$\mathbb{L}_{\begin{smallmatrix}\mathbb{N}\\m\end{smallmatrix}}^+\mathbb{K}=\sum_m\mathbb{L}_{\begin{smallmatrix}\mathbb{N}\\m\end{smallmatrix}}^+\mathbb{K}\in\nabla_{\mathbb{N}}\mathbb{K}\text{ Gel}$$

$$\mathbb{L}_{\begin{smallmatrix}\mathbb{N}\\m\end{smallmatrix}}^+\mathbb{K}=\frac{\mathbb{L}\mathbf{x}\cdots\mathbf{x}\mathbb{L}\stackrel{\text{m-lin}}{\rightarrow}\mathbb{TK}}{\pi\ltimes\mathbb{T}=\mathbb{T}}=\mathbb{K}_m^+\nabla\mathbb{L}_{\begin{smallmatrix}\mathbb{N}\\m\end{smallmatrix}}\mathbb{K}\text{ symm}$$

$$\mathbb{L}_{\begin{smallmatrix}\mathbb{N}\\p+q\end{smallmatrix}}^{p+q}\mathop{\mathbf{x}}\limits_{\mathbb{K}}\mathbb{L}_{\begin{smallmatrix}\mathbb{N}\\p\end{smallmatrix}}\mathbb{K}\mathbf{x}\mathbb{L}_{\begin{smallmatrix}\mathbb{N}\\q\end{smallmatrix}}^q$$

$${}_M\mathbb{L}\left(\mathbb{I}\mathbf{x}\mathbb{I}\right)=\sum_{P\subset M}^{\sharp P=p}({}_P\mathbb{L})\left({}_{M\setminus P}\mathbb{L}\mathbb{I}\right)$$

$$\left[\begin{smallmatrix} \mathbb{L} \\ + \\ m\mathbb{L} \end{smallmatrix}\right] \mathbb{T} \mathbf{x} \mathbb{T}' = \sum_{1 \leq \nu_1 < \cdots < \nu_p \leq m} \left[\begin{smallmatrix} \nu_1 \mathbb{L} \\ + \\ \nu_p \mathbb{L} \end{smallmatrix}\right] \mathbb{T} \left[\begin{smallmatrix} \nu_{p+1} \mathbb{L} \\ + \\ \nu_m \mathbb{L} \end{smallmatrix}\right] \mathbb{T}'$$

$$\mathbb{K}_m^+ \mathbb{L}_{\mathbb{K}\Delta}^+ \overset{?}{\longleftarrow} \mathbb{L}_{\Delta\mathbb{K}}^+ \mathbb{K}_m^+$$

$$_{i\in I}\mathbf{x}_{\mathbb{L}}\big|_{j\in J}\mathbb{T}^j=\text{Per}\,\mathbb{L}_{\mathbb{T}}^j$$

$$\text{LHS} \; = m! \, _{i\in I}\mathbf{x}_{\mathbb{L}}\big|_{p_{+j}\mathbf{x}_{\mathbb{J}}}\mathbb{T}^j \; = \sum_{I\stackrel{\sim}{\rightarrow}\pi J} \, _{i\in I}\mathbf{x}_{\mathbb{L}}\mathbb{L}_{i\in I}\mathbb{T}^{\pi_i} \; = \sum_{\pi}\prod_{i\in I}\mathbb{L}_{\mathbb{T}}^{\pi_i} \; = \text{RHS}$$

$$\mathbb{L}_{\Delta\mathbb{I}}^+\mathbb{I}=\sum_m\mathbb{L}_{\Delta\mathbb{I}}^+\mathbb{I}^m$$

$$\mathbb{L}_{\Delta\mathbb{I}}^+\mathbb{I}^m=\left\{\mathbb{L}\mathbf{x}\ldots\mathbf{x}\mathbb{L}\overset{\text{m-lin}}{\longrightarrow}\mathbb{T}\mathbb{I}\right\}_{\pi\ltimes\mathbb{T}=\mathbb{T}}=\mathbb{K}_m^+\mathbb{L}_{\Delta\mathbb{I}}^+\mathbb{I}=\underbrace{\mathbb{L}_{\Delta\mathbb{K}}^+\mathbb{K}}_{\mathbf{x}\mathbb{I}}\mathbb{I}$$

$$\mathbb{L}_{\Delta\mathbb{I}}^{\prime\,p+q}\mathbf{x}\mathbb{L}_{\Delta\mathbb{I}}^p\mathbb{I}\mathbf{x}\mathbb{L}_{\Delta\mathbb{I}}^{\prime\,q}\mathbb{I}$$

$$\mathbb{L}_{\Delta\mathbb{I}}^+\mathbb{I}=\sum_m\mathbb{L}_{\Delta\mathbb{I}}^+\mathbb{I}^m\in\mathbb{N}\mathbb{K}\text{ Gel}$$

$$\mathbb{L}_{\Delta\mathbb{K}}^+\overset{\mathbb{C}}{\underset{p_+}{\rightleftarrows}}\mathbb{L}_{\Delta\mathbb{K}}^{\mathbb{N}}$$

$$p_+\mathbb{T}=\frac{1}{m!}\sum_{\pi\in\mathbb{C}|m}\pi\ltimes\mathbb{T}$$

$$p_+\left(\tau\ltimes\mathbb{T}\right)=p_+\mathbb{T}$$

$$p_+\left(p_+\mathbb{T}\right)=p_+\mathbb{T}$$

$$\mathbb{L}_{\Delta\mathbb{I}}^+\overset{\mathbb{C}}{\underset{p_+}{\rightleftarrows}}\mathbb{L}_{\Delta\mathbb{I}}^{\mathbb{N}}$$

$$\left(p_+\mathbb{T}\right)\mathbf{x}\left(p_+\mathring{\mathbb{T}}\right)=\left[\begin{smallmatrix}m\\pq\end{smallmatrix}\right]p_+\left(\mathbb{T}\mathbf{x}\mathring{\mathbb{T}}\right)$$

$$\begin{aligned} m! \mathrel{\mathcal{M}}\!\!\! \perp\!\!\!\perp p_+\left(\mathbb{T}\mathbf{x}\mathring{\mathbb{T}}\right) &= \sum_{\pi \in \mathbb{C}|M} \pi_M \mathbb{L}\left(\mathbb{T}\mathbf{x}\mathring{\mathbb{T}}\right) = \sum_{\pi \in \mathbb{C}|M} \left(\pi_{M_{<}} \mathbb{L}\mathbb{T}\right) \times \left(\pi_{M_{>}} \mathbb{L}\mathring{\mathbb{T}}\right) \\ &= \sum_{|P|=p} \sum_{M_{<} \xrightarrow{\sim} \sigma P} (\mathcal{M}\mathbb{T}) \times \sum_{M_{>} \xrightarrow{\sim} \tau M \mathrel{\mathcal{M}}\!\!\! \perp\!\!\!\perp P} \left(\mathcal{L}\mathring{\mathbb{T}}\right) = \sum_{|P|=p} p! \left(\mathcal{L}\!\!\! \perp\!\!\!\perp p_+\mathbb{T}\right) \times q! \left(\mathrel{\mathcal{M}}\!\!\! \perp\!\!\!\perp p_+\mathring{\mathbb{T}}\right) \\ p_+\left(\left(p_+\mathbb{T}\right)\mathbf{x}\mathring{\mathbb{T}}\right) &= p_+\left(\mathbb{T}\mathbf{x}\mathring{\mathbb{T}}\right) = p_+\mathbb{T}\mathbf{x}\left(p_+\mathring{\mathbb{T}}\right) \\ \left[\begin{smallmatrix}m\\pq\end{smallmatrix}\right]p_+\left(\left(p_+\mathbb{T}\right)\mathbf{x}\mathring{\mathbb{T}}\right) &= p_+\left(\left(p_+\mathbb{T}\right)\mathbf{x}\left(p_+\mathring{\mathbb{T}}\right)\right) = \left(p_+\mathbb{T}\right)\mathbf{x}\left(p_+\mathring{\mathbb{T}}\right) = \left[\begin{smallmatrix}m\\pq\end{smallmatrix}\right]p_+\mathbb{T}\mathbf{x}\left(p_+\mathring{\mathbb{T}}\right) \end{aligned}$$

$$\mathbb{L}_{\triangleleft \mathbb{K}}^+ \overset{\mathbb{N}}{\mathbb{K}} \text{ assoc comm}$$

$$\begin{aligned} \left(\mathbb{T}\mathbf{x}\mathring{\mathbb{T}}\right)\mathbf{x}\mathring{\mathbb{T}} &= \left[\begin{smallmatrix}m\\p+q:r\end{smallmatrix}\right]p_+\left(\left(\mathbb{T}\mathbf{x}\mathring{\mathbb{T}}\right)\mathbf{x}\mathring{\mathbb{T}}\right) = \left[\begin{smallmatrix}m\\p:q:r\end{smallmatrix}\right]p_+p_+\left(\mathbb{T}\mathbf{x}\mathring{\mathbb{T}}\right)\mathbf{x}\mathring{\mathbb{T}} = \left[\begin{smallmatrix}m\\p:q:r\end{smallmatrix}\right]p_+\left(\mathbb{T}\mathbf{x}\mathring{\mathbb{T}}\mathbf{x}\mathring{\mathbb{T}}\right) \\ \mathbb{T}\mathbf{x}\mathring{\mathbb{T}} &= \left[\begin{smallmatrix}m\\pq\end{smallmatrix}\right]p_+\left(\mathbb{T}\mathbf{x}\mathring{\mathbb{T}}\right) = \left[\begin{smallmatrix}m\\pq\end{smallmatrix}\right]p_+\tau \ltimes \left(\mathring{\mathbb{T}}\mathbf{x}\mathbb{T}\right) = \left(\mathring{\mathbb{T}}\mathbf{x}\mathbb{T}\right) \end{aligned}$$

$$\tau\left[\begin{smallmatrix}M_{<}\\M_{>}\end{smallmatrix}\right]=\left[\begin{smallmatrix}M_{>}\\M_{<}\end{smallmatrix}\right]$$

$$\mathring{\mathbb{T}}\mathbf{x}\cdots\mathbf{x}\mathring{\mathbb{T}}^k=\left[\begin{smallmatrix}m\\p_0\cdotp\cdotp p_k\end{smallmatrix}\right]p_+\mathring{\mathbb{T}}\mathbf{x}\cdots\mathbf{x}\mathring{\mathbb{T}}^k$$

$$\text{LHS} \, = \, \mathring{\mathbb{T}}\mathbf{x}\Big(\mathring{\mathbb{T}}\mathbf{x}\cdots\mathbf{x}\mathring{\mathbb{T}}^k\Big) = \left[\begin{smallmatrix}m\\p_0:p_1\cdotp\cdotp p_k\end{smallmatrix}\right]p_+\mathring{\mathbb{T}}\mathbf{x}\Big(\mathring{\mathbb{T}}\mathbf{x}\cdots\mathbf{x}\mathring{\mathbb{T}}^k\Big) = \left[\begin{smallmatrix}m\\p_0:p_1\cdotp\cdotp p_k\end{smallmatrix}\right]\left[\begin{smallmatrix}p_1+\cdots+p_k\\p_1\cdotp\cdotp p_k\end{smallmatrix}\right]p_+\mathring{\mathbb{T}}\mathbf{x}\Big(p_+\mathring{\mathbb{T}}\mathbf{x}\cdots\mathbf{x}\mathring{\mathbb{T}}^k\Big) =$$

$$\mathbb{L}_{\triangleleft \mathbb{K}}^+ \overset{m}{\mathbb{K}} = \mathbb{L}_{\triangleleft \mathbb{K} \triangleleft \mathbb{K}}^+ \overset{m}{\mathbb{K}} \ni \mathbb{T}' = \mathop{\mathbf{x}}\limits_{j \in J} \mathring{\mathbb{T}}^j$$

$$\left(\mathbf{x}\mathbb{L}\right)\mathbf{x}\left(\mathbf{x}\mathring{\mathbb{L}}\right)\mathop{:=}\mathbb{L}\mathbf{x}\mathring{\mathbb{L}}$$

$$(\mathfrak{X}\mathbb{L})\mathfrak{X}\mathbb{T}=\mathbb{L}\mathbb{T}$$

$$\mathfrak{X}\mathbb{L}=\mathop{\mathfrak{X}}\limits_{i\in I}(\mathfrak{X}\mathbb{L}_i)$$

$$(\mathfrak{X}\mathbb{L})\mathfrak{X}\Big(\mathfrak{X}\mathbb{L}'\Big)=\mathbb{L}\mathfrak{X}\mathbb{L}'$$

$$\mathbb{T}\mathfrak{X}\mathbb{T}'=\Big(\mathop{\mathfrak{X}}\limits_{i\in I}\mathbb{T}^i\Big)\mathfrak{X}\Big(\mathop{\mathfrak{X}}\limits_{j\in J}\mathbb{T}'^j\Big)=\text{Per}\,\,\Big(\mathbb{T}^i\mathfrak{X}\mathbb{T}'^j\Big)$$

$$\Big(\mathop{\mathfrak{X}}\limits_{i\in I}\mathbb{L}_i\Big)\mathfrak{X}\Big(\mathop{\mathfrak{X}}\limits_{j\in J}\mathbb{L}'_j\Big)=\text{Per}\,\,\Big(\mathbb{L}_i\mathfrak{X}\mathbb{L}'_j\Big)=\text{Per}\,\,\Big(\mathbb{L}_i\Big(\mathfrak{X}\mathbb{L}'_j\Big)\Big)=\Big(\mathop{\mathfrak{X}}\limits_{i\in I}\mathbb{L}_i\Big)\Big(\mathop{\mathfrak{X}}\limits_{j\in J}\Big(\mathfrak{X}\mathbb{L}'_j\Big)\Big)$$

$$(\mathfrak{X}(\mathfrak{X}\mathbb{L}_i))\mathfrak{X}\Big(\mathfrak{X}\mathbb{T}^j\Big)=(\mathfrak{X}(\mathfrak{X}\mathbb{L}_i))\mathfrak{X}\Big(\mathfrak{X}\mathbb{T}^j\Big)=(\mathfrak{X}\mathbb{L}_i)\Big(\mathfrak{X}\mathbb{T}^j\Big)=\text{Per}\,\,\Big(\mathbb{L}_i\mathbb{T}^j\Big)=\text{Per}\,\,\Big((\mathfrak{X}\mathbb{L}_i)\mathfrak{X}\mathbb{T}^j\Big)$$

$$\mathbb{K}_{\mathbb{N}}\mathbb{N}\mathbb{L}=\sum_m\mathbb{K}_{\mathbb{M}}\mathbb{N}\mathbb{L}\in\mathbb{K}\mathbb{N}\text{-abel}$$

$$\begin{array}{ccc} \mathbb{K}_{\mathbb{N}}\mathbb{N}\mathbb{L} & & \\ \uparrow & \searrow \scriptstyle \tilde{\varphi} & \\ & \mathbb{V} & \\ \mathbb{L} & \nearrow \scriptstyle \varphi & \mathbb{L}\in\mathbb{K}\mathbb{N}\text{ Gel} \end{array}$$

$$\mathbb{L}\varphi\mathfrak{X}\mathbb{L}'\varphi=0$$

$$\mathbb{L}\iota=\mathbb{L}+\mathcal{J}$$

$$\mathbb{L}\iota\mathfrak{X}\mathbb{L}'\iota=0$$

$$(\mathbb{L}\mathfrak{X}\cdots\mathfrak{X}_m\mathbb{L}+\mathcal{J})\,\tilde{\varphi}:=\mathbb{L}\varphi\mathfrak{X}\cdots\mathfrak{X}_m\mathbb{L}\varphi$$

$$(\mathbb{L}\mathfrak{X}\cdots\mathfrak{X}_m\mathbb{L}+\mathcal{J})\,\mathbb{K}_{\mathbb{N}}\mathbb{N}\mathbb{L}:=\mathbb{L}\mathbb{L}\mathfrak{X}\cdots\mathfrak{X}_m\mathbb{L}\mathbb{L}+\mathcal{J}$$

$$\mathbb{K}_{\mathbb{N}}\mathbb{N}\mathbb{L}:=\mathbb{K}_{\mathbb{N}}\mathbb{N}\mathbb{L}+\mathbb{K}_{\mathbb{N}}\mathbb{N}\mathbb{L}\mathbb{L}\mathfrak{X}\mathbb{L}'+\mathbb{L}'\mathfrak{X}\mathbb{L}\mathbb{K}_{\mathbb{N}}\mathbb{N}\mathbb{L}$$

$$\mathbb{K}_{\overline{m}}\mathbb{N}\mathbb{L}:=\mathbb{K}_{\overline{m}}\mathbb{N}\mathbb{L}+\mathbb{K}_{\pi_1}\mathbb{L}\mathfrak{X}\cdots\mathfrak{X}_{\pi_1}\mathbb{L}-\mathop{-}^{\pi}1\mathbb{L}\mathfrak{X}\cdots\mathfrak{X}_m\mathbb{L}$$

$$\mathbb{L} = \mathop{\mathbb{X}}\limits_{i \in I} \mathbb{L} = \left[\begin{array}{c} \mathbb{L}_{i_1} \\ + \\ \mathbb{L}_{i_m} \end{array} \right]$$

$$I=\left\{i_1<\cdots <i_m\right\} \text{ geordnet}$$

$$\mathbb{K}_{\mathbb{N}}\nabla\mathbb{L}\in\mathbb{K}\nabla_{\substack{1:1\\0}}\mathbb{L}$$

$$\mathbb{L}\star\mathbb{J}'=\mathop{\mathbb{X}}\limits_{i\in I}\mathbb{L}\star\mathop{\mathbb{X}}\limits_{j\in J}\mathbb{J}'=\det\mathbb{L}\star\mathbb{J}'=\det\mathbb{L}\eta\left(\mathbb{J}'^*\right)$$

$$\mathbb{K}_{\mathbb{M}}\nabla\mathbb{L}\rightarrow\mathbb{K}\nabla\mathbb{L}_{\triangleleft\mathbb{K}}^m$$

$$\mathop{\mathbb{X}}\limits_{i\in I}\mathbb{L}_i|\mathop{\mathbb{X}}\limits_{j\in J}\mathbb{T}^j=\det\mathbb{L}_i\mathbb{T}^j$$

$$\text{LHS} \, = m! \mathop{\mathbb{X}}\limits_{i \in I} \mathbb{L}_i | p_- \mathop{\mathbb{X}}\limits_{j \in J} \mathbb{T}^j = \sum_{I \stackrel{\sim}{\rightarrow} \pi J} -^{\pi} 1_{\mathop{\mathbb{X}}\limits_{i \in I} \mathbb{L}_i \mathop{\mathbb{X}}\limits_{i \in I} \mathbb{T}^{\pi_i}} = \sum_{\pi} \prod_{i \in I} \mathbb{L}_i \mathbb{T}^{\pi_i} = \text{RHS}$$

$$\mathbb{L}_{\mathbb{N}}\nabla\mathbb{L}=\mathbb{L}\mathbb{X}\mathbb{K}_{\mathbb{N}}\nabla\mathbb{L}$$

$$_N\mathbb{L}\in\mathbb{K}_{\mathfrak{r}}\nabla\mathbb{L}$$

$$_N\mathbb{L}\,\mathbb{L}=\, _N\mathbb{L}\,\det\,\mathbb{L}$$