

$$\mathbb{L}_{\mathbb{K}^m} \xleftarrow{\mathbb{L} \mathbb{I} \leftarrow \mathbb{L} : \mathbb{I}} \mathbb{L}_{\mathbb{K}^t} \ltimes \mathbb{L}_{\mathbb{K}^m}$$

$${}_M\mathsf{L}(\mathbb{L}\mathbb{I})=(\mathsf{L}\mathbb{L})\mathbb{I}$$

$$\begin{bmatrix} j_1 \\ + \\ j_m \end{bmatrix} \mathbb{L}_{\mathbb{K}^m} \mathbb{I} = \begin{bmatrix} j_1 \\ + \\ j_m \end{bmatrix} \mathbb{I}$$

$$\mathbb{L}\mathbb{I}\mathbb{I}=\mathbb{L}\mathbb{I}\mathbb{I}$$

$$\mathbb{L}_{\mathbb{K}^m}(\mathbb{I}\mathbf{x}\mathbb{I})=\mathbb{L}_{\mathbb{K}^m}\mathbb{I}\mathbf{x}\mathbb{L}_{\mathbb{K}^m}\mathbb{I}$$

$$\mathbb{L}\ltimes\mathbb{I}=\partial_t^0(\exp t\mathbb{L})\mathbb{I}$$

$${}_M\mathsf{L}(\mathbb{L}\ltimes\mathbb{I})=\sum_i^M\begin{bmatrix} \mathbb{L} & \mathbb{L} \\ M\vdash i & \end{bmatrix}\mathbb{I}$$

$$\begin{bmatrix} \mathbb{L} \\ + \\ m \end{bmatrix} \overbrace{\mathbb{L}_{\mathbb{K}^m}\mathbb{I}}^{\text{sym}} = \mathbb{L}\left(\begin{bmatrix} \mathbb{L} & \mathbb{L} \\ + & \\ \mathbb{L}v & \end{bmatrix}\mathbb{I}\right)$$

$$\mathbb{L}_{\mathbb{K}^{p+q}}\mathbb{I}\mathbf{x}\mathbb{I}=\mathbb{L}_{\mathbb{K}^p}\mathbb{I}\mathbf{x}\mathbb{L}_{\mathbb{K}^q}\mathbb{I}$$

$$\mathbb{L}_{\mathbb{K}^m} \xleftarrow{\mathbb{L} \mathbb{I} \leftarrow \mathbb{L} : \mathbb{I}} \mathbb{L}_{\mathbb{K}^t} \ltimes \mathbb{L}_{\mathbb{K}^m}$$

$${}_M\mathsf{L}(\mathbb{L}\mathbb{I})=(\mathsf{L}\mathbb{L})\mathbb{I}$$

$$\begin{bmatrix} j_1 \\ + \\ j_m \end{bmatrix} \mathbb{L}_{\mathbb{K}^m} \mathbb{I} = \begin{bmatrix} j_1 \\ + \\ j_m \end{bmatrix} \mathbb{I}$$

$$\mathbb{L}\mathbb{I}\mathbb{I}=\mathbb{L}\mathbb{I}\mathbb{I}$$

$$\mathbb{L}_{\mathbb{K}^m}(\mathbb{I}\mathbf{x}\mathbb{I})=\mathbb{L}_{\mathbb{K}^m}\mathbb{I}\mathbf{x}\mathbb{L}_{\mathbb{K}^m}\mathbb{I}$$

$$\mathbb{L}\ltimes\mathbb{I}=\partial_t^0(\exp t\mathbb{L})\mathbb{I}$$

$${}_M\mathsf{L}(\mathbb{L}\ltimes\mathbb{I})=\sum_i^M{}_M\dot{\vee}_i\begin{bmatrix} \mathbb{L} & \mathbb{L} \\ M\vdash i & \end{bmatrix}\mathbb{I}$$

$$\begin{bmatrix} \text{L} \\ \text{+} \\ m \end{bmatrix} \xrightarrow{\text{symm}} \text{L}_{\text{+}}^m = \text{L} \left(\begin{bmatrix} \text{L} & \text{L} \\ \text{+} & v \end{bmatrix} \right)$$

$$\underbrace{\text{L}_{\text{+}}^{p+q}}_{\text{L}_{\text{+}}^p \text{ L}^q} \text{ L} \times \text{L} = \underbrace{\text{L}_{\text{+}}^p}_{\text{L}_{\text{+}}^p \text{ L}^q} \text{ L} \times \underbrace{\text{L}_{\text{+}}^q}_{\text{L}_{\text{+}}^q \text{ L}^p}$$

$$\begin{array}{ccc} \Psi|\text{L} & \xrightarrow{a} & \Psi|\text{L}_{\text{+}}^{\mathbb{N}} \\ b \downarrow & & \downarrow s \\ n\mathbb{K}^n & \xrightarrow{q} & \begin{bmatrix} n+m+1 \\ n+m \\ m \end{bmatrix} \mathbb{K} \end{array}$$

$$\begin{array}{ccc} \text{L}_{\text{+}}^m & \xleftarrow{\text{L}} & \text{L}_{\text{+}}^m \\ b \downarrow & & \downarrow s \\ \begin{bmatrix} n+m+1 \\ n+m \\ m \end{bmatrix} \mathbb{K} & \xleftarrow[\text{per } {}_I \text{L} \text{ L}^J]{} & \begin{bmatrix} n \\ m \end{bmatrix} \mathbb{K} \end{array}$$

$$\begin{bmatrix} n+m+1 \\ n+m \\ m \end{bmatrix} \stackrel{I}{\leftarrow} \left(\sum_{|J|=m} \underbrace{\text{per } {}_I \text{L} \text{ L}^J \text{ L}^J}_{\text{per } {}_I \text{L} \text{ L}^J} \text{ L}^J \right) \leftarrow \begin{bmatrix} J \\ m \end{bmatrix} \stackrel{J}{\leftarrow}$$

$$\begin{array}{ccc}
\Psi|\mathbb{L} & \xrightarrow{a} & \Psi|\overset{\mathbb{N}}{\mathbb{L}} \triangleleft \mathbb{K} \\
b \downarrow & & \downarrow s \\
_n\mathbb{K}^n & \xrightarrow{q} & \begin{bmatrix} n \\ m \end{bmatrix} \mathbb{K}
\end{array}$$

$$\begin{array}{ccc}
\overset{m}{\mathbb{L}} \triangleleft \mathbb{K} & \xleftarrow{\mathbb{L}} & \overset{m}{\mathbb{L}} \triangleleft \mathbb{K} \\
b \downarrow & & \downarrow s \\
\begin{bmatrix} n \\ m \end{bmatrix} \mathbb{K} & \xleftarrow{\det {}_I \mathbb{L} \ \mathbb{L} \ \overset{J}{\mathbb{L}}} & \begin{bmatrix} n \\ m \end{bmatrix} \mathbb{K}
\end{array}$$

$$\begin{bmatrix} n \\ m \end{bmatrix} \xleftarrow{I \left(\sum_{|J|=m} \underbrace{\det {}_I \mathbb{L} \ \mathbb{L} \ \overset{J}{\mathbb{L}}}_J \overset{'}{\mathbb{1}} \right)} \leftarrow \begin{bmatrix} n \\ m \end{bmatrix} \xleftarrow{J \left({}_J \overset{'}{\mathbb{1}} \right)}$$