$$\mathbb{K}^{n} \trianglelefteq \overset{m}{\mathbb{K}} \leftarrow {}_{n}\mathbb{K}^{n} \ltimes \mathbb{K}^{n} \trianglelefteq \overset{m}{\mathbb{K}}$$

$${}_{n}\mathbb{K}^{n} \longrightarrow {}_{\begin{bmatrix} n+m+1 \\ m \end{bmatrix}}\mathbb{K}^{\begin{bmatrix} n+m+1 \\ m \end{bmatrix}}$$

$$.\lfloor^{\cdot} \mapsto \operatorname{per} \;{}_{\bullet}\lfloor^{\bullet}$$

$$.\lfloor^{\cdot} \;.\mathbb{L}^{J} = \sum_{|I|=m} .\mathbb{L}^{I} \operatorname{per} \;{}_{I}\lfloor^{J}$$

$$.\lfloor^{\cdot} \sum_{J} .\mathbb{L}^{J} \;{}_{J}\rceil = \sum_{J} \underbrace{.\lfloor^{\cdot} .\mathbb{L}^{J}}\;{}_{J}\rceil = \sum_{|I|=|J|} .\mathbb{L}^{I} \overbrace{\sum_{J} \operatorname{per} \;{}_{I}\lfloor^{J} \;{}_{J}\rceil}$$

$$.\lfloor^{\cdot} \;\underbrace{.\acute{\lfloor}^{\cdot} .\rceil} = \underbrace{.\lfloor^{\cdot} .\acute{\lfloor}^{\cdot}}\,.\rceil$$

$$\operatorname{per} \;{}_{I}\underbrace{.\lfloor^{\cdot} .\acute{\lfloor}^{\cdot}}^{K} = \sum_{\pi} {}_{i_1}\lfloor^{\cdot} \;.\acute{\lfloor}^{k_{\pi_1}} \cdots {}_{i_m}\lfloor^{\cdot} \;.\acute{\lfloor}^{k_{\pi_m}} = \sum_{j_1 \cdots j_m} \sum_{\pi} {}_{i_1}\lfloor^{j_1} \;{}_{j_1}\acute{\lfloor}^{k_{\pi_1}} \cdots {}_{i_m}\lfloor^{j_m} \;{}_{j_m}\acute{\lfloor}^{k_{\pi_m}}$$

$$= \sum_{j_1 \cdots j_m \text{ dist}} \operatorname{per} \left({}_{i_\mu}\lfloor^{j_\mu} \;{}_{j_\mu}\acute{\lfloor}^{k_\nu} \right) = \sum_{j_1 \leqslant \cdots \leqslant j_m} \sum_{\sigma:\pi} {}_{i_1}\lfloor^{j_{\sigma_1}} \;{}_{j_{\sigma_1}}\acute{\lfloor}^{k_{\pi_1}} \cdots {}_{i_m}\lfloor^{j_{\sigma_m}} \;{}_{j_{\sigma_m}}\acute{\lfloor}^{k_{\pi_m}}$$

$$= \sum_{j_1 \leqslant \cdots \leqslant j_m} \overbrace{\sum_{\sigma} \overset{\sigma}{-1} \;{}_{i_1}\lfloor^{j_{\sigma_1}} \cdots {}_{i_m}\lfloor^{j_{\sigma_m}}} \overbrace{\sum_{\tau} {}_{j_1}\acute{\lfloor}^{k_{\tau_1}} \cdots {}_{j_m}\acute{\lfloor}^{k_{\tau_m}}} = \sum_{J} \operatorname{per} \;{}_{I}\lfloor^{J} \operatorname{per} \;{}_{J}\acute{\lfloor}^{K}$$

$$\tau = \pi \ltimes \sigma^{-1}$$

$${}_{j_{\sigma_i}}\acute{\lfloor}^{k_{\pi_i}} = {}_{j_\mu}\acute{\lfloor}^{k_{\tau_\mu}} \Leftarrow \tau\left(\sigma_i\right) = \pi_i$$

$$.\lfloor^{\cdot} \;\underbrace{.\rceil \bowtie .\acute{\rceil}} = \underbrace{.\lfloor^{\cdot} .\rceil} \bowtie \underbrace{.\lfloor^{\cdot} .\acute{\rceil}}$$

$$\underbrace{.\lfloor^{\cdot} \mathbb{L}^{k_1}} \bowtie \cdots \bowtie \underbrace{.\lfloor^{\cdot} \mathbb{L}^{k_m}} = \sum_{j_1 \cdots j_m} \overbrace{.\mathbb{L}^{j_1} \;{}_{j_1}\lfloor^{k_1}} \bowtie \cdots \bowtie \overbrace{.\mathbb{L}^{j_m} \;{}_{j_m}\lfloor^{k_m}}$$

$$= \sum_{|J|=m} .\mathbb{L}^{J} \sum_{\pi} {}_{j_1}\lfloor^{k_{\pi_1}} \cdots {}_{j_m}\lfloor^{k_{\pi_m}} = \sum_{|J|=m} .\mathbb{L}^{J} \operatorname{per} \;{}_{J}\lfloor^{K} = .\lfloor^{\cdot} \;.\mathbb{L}^{K}$$

$$\mathbb{K}^{n} \trianglelefteq \overset{m}{\mathbb{K}} \overset{?}{\leftarrow} {}_{n}\mathbb{K}^{n} \ltimes \mathbb{K}^{n} \trianglelefteq \overset{m}{\mathbb{K}}$$

$${}_{n}\mathbb{K}^{n} \longrightarrow {}_{\begin{bmatrix} n \\ m \end{bmatrix}}\mathbb{K}^{\begin{bmatrix} n \\ m \end{bmatrix}}$$

$$\llcorner^{\cdot} \mapsto \det \, {}_{\bullet}\llcorner^{\bullet}$$

$$\llcorner^{\cdot} \;\; \mathbb{L}^{J} = \sum_{|I|=m} \mathbb{L}^{I} \det \; {}_{I}\llcorner^{J}$$

$$\llcorner^{\cdot} \; \sum_{J} \mathbb{L}^{J} \; {}_{J}\urcorner = \sum_{J} \underbrace{\llcorner^{\cdot} \mathbb{L}^{J}} \; {}_{J}\urcorner = \sum_{|I|=|J|} \mathbb{L}^{I} \overbrace{\sum_{J} \det \; {}_{I}\llcorner^{J} \; {}_{J}\urcorner}$$

$$\llcorner^{\cdot} \;\; \underbrace{\acute{\llcorner} \urcorner} = \underbrace{\llcorner^{\cdot} \acute{\llcorner}^{\cdot}} \urcorner$$

$$\det \; {}_{I}\underbrace{\llcorner^{\cdot} \acute{\llcorner}^{\cdot}}^{K} = \sum_{\pi} \overset{\pi}{-1} \;\; {}_{i_1}\llcorner^{\cdot} \;\; \acute{\llcorner}^{k_{\pi_1}} \cdots \;\; {}_{i_m}\llcorner^{\cdot} \;\; \acute{\llcorner}^{k_{\pi_m}} = \sum_{j_1 \cdots j_m} \sum_{\pi} \overset{\pi}{-1} \;\; {}_{i_1}\llcorner^{j_1} \;\; {}_{j_1}\acute{\llcorner}^{k_{\pi_1}} \cdots \;\; {}_{i_m}\llcorner^{j_m} \;\; {}_{j_m}\acute{\llcorner}^{k_{\pi_m}}$$

$$= \sum_{j_1 \cdots j_m \text{ dist}} \det \left({}_{i_\mu}\llcorner^{j_\mu} \; {}_{j_\mu}\acute{\llcorner}^{k_\nu} \right) = \sum_{j_1 < \cdots < j_m} \sum_{\sigma:\pi} \overset{\pi}{-1} \;\; {}_{i_1}\llcorner^{j_{\sigma_1}} \;\; {}_{j_{\sigma_1}}\acute{\llcorner}^{k_{\pi_1}} \cdots \;\; {}_{i_m}\llcorner^{j_{\sigma_m}} \;\; {}_{j_{\sigma_m}}\acute{\llcorner}^{k_{\pi_m}}$$

$$= \sum_{j_1 < \cdots < j_m} \overbrace{\sum_{\sigma} \overset{\sigma}{-1} \; {}_{i_1}\llcorner^{j_{\sigma_1}} \cdots \; {}_{i_m}\llcorner^{j_{\sigma_m}}} \overbrace{\sum_{\tau} \overset{\tau}{-1} \; {}_{j_1}\acute{\llcorner}^{k_{\tau_1}} \cdots \; {}_{j_m}\acute{\llcorner}^{k_{\tau_m}}} = \sum_{J} \det \; {}_{I}\llcorner^{J} \; \det \; {}_{J}\acute{\llcorner}^{K}$$

$$\tau = \pi \ltimes \sigma^{-1}$$

$$\overset{\sigma}{-1} \; \overset{\tau}{-1} = \overset{\pi}{-1}$$

$$ {}_{j_{\sigma_i}}\acute{\llcorner}^{k_{\pi_i}} = {}_{j_\mu}\acute{\llcorner}^{k_{\tau_\mu}} \Leftarrow \tau\left(\sigma_i\right) = \pi_i$$

$$\llcorner^{\cdot} \;\; \underbrace{\urcorner \boxtimes \acute{\urcorner}} = \underbrace{\llcorner^{\cdot} \urcorner} \boxtimes \underbrace{\llcorner^{\cdot} \acute{\urcorner}}$$

$$\underbrace{\llcorner^{\cdot} \mathbb{L}^{k_1}} \boxtimes \cdots \boxtimes \underbrace{\llcorner^{\cdot} \mathbb{L}^{k_m}} = \sum_{j_1 \cdots j_m} \overbrace{\mathbb{L}^{j_1} \;\; {}_{j_1}\llcorner^{k_1}} \boxtimes \cdots \boxtimes \overbrace{\mathbb{L}^{j_m} \;\; {}_{j_m}\llcorner^{k_m}}$$

$$= \sum_{|J|=m} \mathbb{L}^{J} \sum_{\pi} \overset{\pi}{-1} \;\; {}_{j_1}\llcorner^{k_{\pi_1}} \cdots \;\; {}_{j_m}\llcorner^{k_{\pi_m}} = \sum_{|J|=m} \mathbb{L}^{J} \det \; {}_{J}\llcorner^{K} = \llcorner^{\cdot} \;\; \mathbb{L}^{K}$$