

$$\mathbb{L}_{\triangleleft \mathbb{K}}^m \qquad \mathbb{L}_{\triangleleft \mathbb{K}}^{\mathbb{N}}$$

$$\begin{array}{ccc} \downarrow & & \downarrow \\ \textcolor{blue}{I}\mathbb{L} & & \textcolor{blue}{.}\mathbb{L} \end{array}$$

$$\begin{array}{ccc} \downarrow & & \downarrow \\ \left[\begin{array}{c} n+m+1 \\ m \end{array} \right] \mathbb{K} & & 2^n \mathbb{K} \end{array}$$

$$\left[\begin{array}{c} n+m+1 \\ m \end{array} \right] \mathbb{K} \ni \textcolor{blue}{I}\mathbb{L} \textcolor{blue}{\vdash} \textcolor{blue}{\leftarrow} \mathbb{I} \in \mathbb{L}_{\triangleleft \mathbb{K}}^m$$

$$\mathbb{I} = \sum_{|I|=m} \textcolor{blue}{I}^I \textcolor{blue}{\underbrace{\mathbb{I}\mathbb{I}}}$$

$$2^n \mathbb{K} \ni \textcolor{blue}{.}\mathbb{L} \textcolor{blue}{\vdash} \textcolor{blue}{\nmid} \mathbb{I} \in \mathbb{L}_{\triangleleft \mathbb{K}}^{\mathbb{N}}$$

$$\mathbb{I} = \textcolor{blue}{I}^{\bullet} \textcolor{blue}{\underbrace{\mathbb{I}\mathbb{I}}}$$

$$\mathbb{K}^n_{\triangleleft \mathbb{K}} = \frac{\textcolor{blue}{.}\mathbb{L}^M \text{ free}}{M \subset n: \quad |M|=m} \mathbb{K} \ni \textcolor{blue}{.}\mathbb{I} = \sum_{|M|=m} \textcolor{blue}{.}\mathbb{L}^M \textcolor{blue}{\textcolor{brown}{M}}\mathbb{I}$$

$$\textcolor{blue}{M}\mathbb{I} \in \mathbb{K} \text{ eind}$$

$$\textcolor{blue}{\underbrace{\textcolor{blue}{I}\textcolor{blue}{\textcolor{brown}{X}}\textcolor{blue}{I}}}\textcolor{blue}{\textcolor{brown}{X}}\textcolor{blue}{\textcolor{brown}{I}} = \textcolor{blue}{.}\textcolor{blue}{\textcolor{brown}{X}}\textcolor{blue}{\underbrace{\textcolor{blue}{I}\textcolor{blue}{\textcolor{brown}{X}}\textcolor{blue}{I}}}$$

$$\textcolor{blue}{I}\textcolor{blue}{\textcolor{brown}{X}}\textcolor{blue}{*}\mathbb{I} = \left(\textcolor{blue}{I}\textcolor{blue}{\textcolor{brown}{X}}\textcolor{blue}{I}\right) \textcolor{blue}{I}^N$$

$$\textcolor{blue}{I}\textcolor{blue}{\textcolor{brown}{X}}\textcolor{blue}{I} = {}_N\textcolor{blue}{I} \textcolor{blue}{\underbrace{\textcolor{blue}{I}\textcolor{blue}{\textcolor{brown}{X}}\textcolor{blue}{I}}}$$

$$\textcolor{blue}{.}\textcolor{blue}{\textcolor{brown}{X}}\textcolor{blue}{I} = \overset{pq}{-1} \textcolor{blue}{I}\textcolor{blue}{\textcolor{brown}{X}}\textcolor{blue}{.}\textcolor{blue}{I}$$

$$\textcolor{blue}{I}^I \textcolor{blue}{\textcolor{brown}{X}}\textcolor{blue}{*}\textcolor{blue}{\underbrace{\mathbb{I}^I}} = \textcolor{blue}{I}^I \textcolor{blue}{\textcolor{brown}{X}} \textcolor{blue}{I}^{N \textcolor{brown}{\perp} I} \bigvee_{N \textcolor{brown}{\perp} I}^I \text{ per } \textcolor{blue}{I} \eta^I = {}_N\textcolor{blue}{I} \underbrace{\bigvee_{N \textcolor{brown}{\perp} I}^I \bigvee_{N \textcolor{brown}{\perp} I}^I}_{=1} \textcolor{blue}{I}^I \textcolor{blue}{\textcolor{brown}{X}} \textcolor{blue}{I}^I$$

$$\mathbb{K}^n_{\triangleleft \mathbb{K}} \overset{p+q}{\longleftarrow} \textcolor{blue}{\textcolor{brown}{X}} \mathbb{K}^n_{\triangleleft \mathbb{K}} \textcolor{blue}{\textcolor{brown}{X}} \mathbb{K}^n_{\triangleleft \mathbb{K}} \overset{q}{\longleftarrow}$$

$$\text{bilin}$$

$$\textcolor{blue}{I}^P \textcolor{blue}{\textcolor{brown}{X}} \textcolor{blue}{I}^Q := \bigvee_Q^P \textcolor{blue}{I}^{P \cup Q}$$

$$\varepsilon = \bigvee_B^A \bigvee_C^{A \cup B} = \bigvee_B^A \bigvee_C^A \bigvee_C^B = \bigvee_{B \cup C}^A \bigvee_C^B$$

$$\Rightarrow \underbrace{\mathbb{L}^A \mathbf{x} \mathbb{L}^B} \mathbf{x} \mathbb{L}^C = \varepsilon \cdot \mathbb{L}^{A \cup B \cup C} = \mathbb{L}^A \mathbf{x} \underbrace{\mathbb{L}^B \mathbf{x} \mathbb{L}^C}$$

$$|P>Q||Q>P|=|P||Q|\Rightarrow \overset{P}{\underset{Q}{\vee}} = -1 \overset{pq}{\underset{p}{\vee}} \overset{Q}{\underset{p}{\vee}} \Rightarrow \mathbb{L}^P \mathbf{x} \mathbb{L}^Q = -1 \cdot \mathbb{L}^Q \mathbf{x} \mathbb{L}^P$$

$$\mathbb{L}^M = \mathop{\mathbf{x}}\limits_{j \in M} \mathbb{L}^j = \mathbb{L}^{j_1} \mathbf{x} \cdots \mathbf{x} \mathbb{L}^{j_m}$$

$$M=\left\{j_1\leqslant\cdots\leqslant j_m\right\}$$

$$\begin{array}{ccc} \mathbb{L}_{\triangleleft \mathbb{K}}^m & & \mathbb{L}_{\triangleleft \mathbb{K}}^{\mathbb{N}} \\ \downarrow \scriptstyle \mathbb{L}_I & & \downarrow \scriptstyle \cdot \mathbb{L} \\ \left[\begin{smallmatrix} n \\ m \end{smallmatrix} \right] \mathbb{K} & & 2^n \mathbb{K} \end{array}$$

$$\left[\begin{smallmatrix} n \\ m \end{smallmatrix} \right] \mathbb{K} \ni \mathbb{L}_I \mathbb{T} \leftarrow \mathbb{T} \in \mathbb{L}_{\triangleleft \mathbb{K}}^m$$

$$\mathbb{T} = \sum_{|I|=m} \mathbb{L}^I \underbrace{\mathbb{L} \mathbb{L}}$$

$$2^n \mathbb{K} \ni \cdot \mathbb{L} \mathbb{T} \leftarrow \mathbb{T} \in \mathbb{L}_{\triangleleft \mathbb{K}}^{\mathbb{N}}$$

$$\mathbb{T} = \mathbb{L}^{\bullet} \underbrace{\mathbb{L} \mathbb{L}}$$

$$\mathbb{K}^n_{\triangleleft \mathbb{K}}^m = \frac{\mathbb{L}^M \text{ free}}{M \subset n: \quad |M|=m} \mathbb{K} \ni \cdot \mathbb{T} = \sum_{|M|=m} \mathbb{L}^M \cdot_M \mathbb{T}$$

$${}_M\mathbb{T}\in\mathbb{K}\text{ eind}$$

$$\underbrace{\mathbb{T} \mathbf{x} \mathbb{T}^{\prime} \mathbf{x} \mathbb{T}^{\prime}} = \mathbb{T} \mathbf{x} \underbrace{\mathbb{T}^{\prime} \mathbf{x} \mathbb{T}^{\prime}}$$

$$\mathbb{T}^{\prime} \mathbf{x} \ast \mathbb{T} = \left(\mathbb{T} \mathbf{x} \mathbb{T}^{\prime}\right) \mathbb{L}^N$$

$$\mathbb{T} \mathbf{x} \mathbb{T}^{\prime} = {}_N \mathbb{L} \underbrace{\mathbb{T}^{\prime} \mathbf{x} \ast \mathbb{T}}$$

$$\cdot \mathbb{T} \mathbf{x} \cdot \mathbb{T}^{\prime} = -1 \overset{pq}{\underset{p}{\vee}} \mathbb{T}^{\prime} \mathbf{x} \cdot \mathbb{T}$$

$$\mathbb{L}^I \rtimes \underbrace{\ast \mathbb{L}^I}_{N \lrcorner I} = \mathbb{L}^I \rtimes \mathbb{L}^{N \lrcorner I} \bigvee_{N \lrcorner I}^I \det_I \eta^I = {}_N \mathbb{L} \overbrace{\bigvee_{N \lrcorner I}^I \bigvee_{N \lrcorner I}^I}^{=1} \mathbb{L}^I \rtimes \mathbb{L}^I$$

$$\mathbb{K}^n \begin{smallmatrix} \triangleleft \\ \triangleleft \end{smallmatrix} \mathbb{K}^{p+q} \xleftarrow[\text{bilin}]{\rtimes} \mathbb{K}^n \begin{smallmatrix} \triangleleft \\ \triangleleft \end{smallmatrix} \mathbb{K}^p \rtimes \mathbb{K}^n \begin{smallmatrix} \triangleleft \\ \triangleleft \end{smallmatrix} \mathbb{K}^q$$

$$\mathbb{L}^P \rtimes \mathbb{L}^Q := \bigvee_Q^P \mathbb{L}^{P \cup Q}$$

$$\varepsilon = \bigvee_B^A \bigvee_C^{A \cup B} = \bigvee_B^A \bigvee_C^A \bigvee_C^B = \bigvee_{B \cup C}^A \bigvee_C^B$$

$$\Rightarrow \underbrace{\mathbb{L}^A \rtimes \mathbb{L}^B}_{\varepsilon} \rtimes \mathbb{L}^C = \varepsilon \mathbb{L}^{A \cup B \cup C} = \mathbb{L}^A \rtimes \underbrace{\mathbb{L}^B \rtimes \mathbb{L}^C}_{\varepsilon}$$

$$|P>Q||Q>P|=|P||Q|\Rightarrow \bigvee_Q^P=-1\bigvee_P^Q\Rightarrow \mathbb{L}^P\rtimes\mathbb{L}^Q=-1\mathbb{L}^Q\rtimes\mathbb{L}^P$$

$$\mathbb{L}^M = \bigotimes_{j \in M} \mathbb{L}^j = \mathbb{L}^{j_1} \rtimes \ldots \rtimes \mathbb{L}^{j_m}$$

$$M=\left\{j_1<\cdots <j_m\right\}$$