

$$\mathbb{T}^d = \frac{z \in \mathbb{C}}{|z|=1} = \frac{x+iy \in \mathbb{C}}{x^2+y^2=1}$$

$$\begin{array}{ccc} \mathbb{T}^d & & \\ \uparrow & & \\ \text{hom} & \text{exp} & \\ \downarrow & & \\ i\mathbb{R}^d & & \end{array}$$

$$\begin{array}{ccc} \mathbb{C} = \mathbb{R} + i\mathbb{R} & & \\ \downarrow & & \\ \text{exp} & \text{diff} & \\ \downarrow & & \\ \mathbb{C}^\times = \mathbb{R}_>\mathbb{T} & & \end{array}$$

$$\exp(r+is)={}^re^{is}e={}^re\left({}^s\mathfrak{c}+\Leftrightarrow {}^sL\right)$$

$$\text{Ker }(\exp)=2i\pi\mathbb{Z}=\frac{2\pi in}{\mathbb{Z}\ni n}$$

$$t=r+is \in \text{Ker }(\exp) \Rightarrow 1=\exp(t) \Rightarrow \overline{\exp(t)}=\exp(\Re t)={}^re \Rightarrow r=0$$

$$1=\exp(is)={}^s\mathfrak{c}+\Leftrightarrow {}^sL \Rightarrow 1={}^s\mathfrak{c}$$

$$0={}^s\mathfrak{s} \Rightarrow s=2in\pi$$

$$n\in\mathbb{Z}$$