$$ {}^{p}_{\cup}\mathbb{C}_q \ni a \mapsto \varphi_a \in {}^{p:q}\mathbb{C}^{\cup}_{p:q} $$

$$ {}^{z}\varphi_a = \underbrace{I - a\overset{*}{a}}_{-\,1/2}\,\overbrace{a-z}\,\underbrace{I-\overset{*}{a}z}_{-1}\,\overbrace{I-\overset{*}{a}a}^{1/2} $$

$$ {}^{p}_{\cup}\mathbb{C}_q \xrightarrow[\text{analytic}]{\sigma} {}^{p:q}\mathbb{C}^{\cup}_{p:q} $$

$$ \overline{u{:}v} = \overbrace{1-u\overset{*}{v}}^{-\,1/2}\,\underbrace{z+u}\,\overbrace{1+\overset{*}{v}z}^{-1}\,\overbrace{1-\overset{*}{v}u}^{1/2} $$

$$ = \underbrace{\overbrace{1-u\overset{*}{v}}^{-\,1/2}z+\overbrace{1-u\overset{*}{v}}^{-\,1/2}u}\;\overbrace{\underbrace{1-\overset{*}{v}u}_{-\,1/2}+\underbrace{1-\overset{*}{v}u}_{-\,1/2}\overset{*}{v}z}^{-1} = \begin{array}{c|c} \overbrace{1-u\overset{*}{v}}^{-\,1/2} & \overbrace{1-u\overset{*}{v}}^{-\,1/2}u \\ \hline \underbrace{1-\overset{*}{v}u}_{-\,1/2}\overset{*}{v} & \underbrace{1-\overset{*}{v}u}_{-\,1/2} \end{array}\; z $$

$$ \overline{u{:}v}\,\overline{-u{:}-v} = I $$

$$ \begin{array}{c|c} \overbrace{1-u\overset{*}{v}}^{-\,1/2} & u\overbrace{1-\overset{*}{v}u}^{-\,1/2} \\ \hline \overset{*}{v}\underbrace{1-u\overset{*}{v}}_{-\,1/2} & \underbrace{1-\overset{*}{v}u}_{-\,1/2} \end{array}\; \begin{array}{c|c} \overbrace{1-u\overset{*}{v}}^{-\,1/2} & -\overbrace{1-u\overset{*}{v}}^{-\,1/2}u \\ \hline -\underbrace{1-\overset{*}{v}u}_{-\,1/2}\overset{*}{v} & \underbrace{1-\overset{*}{v}u}_{-\,1/2} \end{array} = \begin{array}{c|c} \overbrace{1-u\overset{*}{v}}^{-1} - u\overbrace{1-\overset{*}{v}u}^{-1}\overset{*}{v} & -\overbrace{1-u\overset{*}{v}}^{-1}u + u\overbrace{1-\overset{*}{v}u}^{-1} \\ \hline \overset{*}{v}\underbrace{1-u\overset{*}{v}}_{-1} - \underbrace{1-\overset{*}{v}u}_{-1}\overset{*}{v} & -\overset{*}{v}\underbrace{1-u\overset{*}{v}}_{-1}u + \underbrace{1-\overset{*}{v}u}_{-1} \end{array} $$

$$ = \begin{array}{c|c} \overbrace{1-u\overset{*}{v}}^{-1} - \overbrace{1-u\overset{*}{v}}^{-1}u\overset{*}{v} & -u\overbrace{1-\overset{*}{v}u}^{-1} + u\overbrace{1-\overset{*}{v}u}^{-1} \\ \hline \underbrace{1-\overset{*}{v}u}_{-1}\overset{*}{v} - \underbrace{1-\overset{*}{v}u}_{-1}\overset{*}{v} & -\underbrace{1-\overset{*}{v}u}_{-1}\overset{*}{v}u + \underbrace{1-\overset{*}{v}u}_{-1} \end{array} = \begin{array}{c|c} \overbrace{1-u\overset{*}{v}}^{-1}\overbrace{1-u\overset{*}{v}} & 0 \\ \hline 0 & \underbrace{1-\overset{*}{v}u}\,\underbrace{1-\overset{*}{v}u}_{-1} \end{array} = \begin{array}{c|c} 1 & 0 \\ \hline 0 & 1 \end{array} $$

$$ \overline{z{:}w} = \begin{array}{c|c} \overbrace{1+\varepsilon z\overset{*}{z}}^{-\,1/2} & \overbrace{1+\varepsilon z\overset{*}{z}}^{-\,1/2}z \\ \hline -\varepsilon\underbrace{1+\varepsilon\overset{*}{z}z}_{-\,1/2}z & \underbrace{1+\varepsilon\overset{*}{z}z}_{-\,1/2} \end{array} $$

$$ {}^{z}\Psi_w = \overbrace{1-w\overset{*}{w}}^{-\,1/2}\,\underbrace{z+w}\,\overbrace{1+\overset{*}{w}z}^{-1}\,\overbrace{1-\overset{*}{w}w}^{1/2} = \underbrace{z+\overbrace{1-w\overset{*}{w}}^{-\,1/2}w}\;\underbrace{\overbrace{1-\overset{*}{w}w}^{-\,1/2}+\overbrace{1-\overset{*}{w}w}^{-\,1/2}\overset{*}{w}z}_{-1} $$

$$ \Psi_w = \begin{array}{c|c} \overbrace{1-w\overset{*}{w}}^{-\,1/2} & \overbrace{1-w\overset{*}{w}}^{-\,1/2}w \\ \hline \overbrace{1-\overset{*}{w}w}^{-\,1/2}\overset{*}{w} & \overbrace{1-\overset{*}{w}w}^{-\,1/2} \end{array} $$

$$ {}^{z}\underline{\Psi_w} - = \overbrace{1-w\overset{*}{w}}^{1/2}\,\overbrace{1+z\overset{*}{w}}^{-1} - \overbrace{1+\overset{*}{w}z}^{-1}\,\overbrace{1-\overset{*}{w}w}^{1/2} $$

$${}^{z}\Phi_w = \overbrace{1-w\overset{*}{w}}^{-\,1/2}\,\underbrace{w-z}_{}\,\overbrace{1-\overset{*}{w}z}^{-1}\,\overbrace{1-\overset{*}{w}w}^{1/2} = w - \overbrace{1-w\overset{*}{w}}^{1/2}\,z\,\overbrace{1-\overset{*}{w}z}^{-1}\,\overbrace{1-\overset{*}{w}w}^{1/2}$$

$$\Phi_w = \begin{array}{c|c} -\overbrace{1-w\overset{*}{w}}^{-\,1/2} & \overbrace{1-w\overset{*}{w}}^{-\,1/2}w \\ \hline -\overbrace{1-\overset{*}{w}w}^{-\,1/2}\overset{*}{w} & \overbrace{1-\overset{*}{w}w}^{-\,1/2} \end{array}$$

$$s_w^\alpha = \Psi_w\, s_0^\alpha\, \Psi_{-w} = \begin{array}{c|c} \overbrace{1-w\overset{*}{w}}^{-\,1/2} & \overbrace{1-w\overset{*}{w}}^{-\,1/2}w \\ \hline \overbrace{1-\overset{*}{w}w}^{-\,1/2}\overset{*}{w} & \overbrace{1-\overset{*}{w}w}^{-\,1/2} \end{array}\; \begin{array}{c|c} \alpha & 0 \\ \hline 0 & 1 \end{array}\; \begin{array}{c|c} \overbrace{1-w\overset{*}{w}}^{-\,1/2} & -\overbrace{1-w\overset{*}{w}}^{-\,1/2}w \\ \hline -\overbrace{1-\overset{*}{w}w}^{-\,1/2}\overset{*}{w} & \overbrace{1-\overset{*}{w}w}^{-\,1/2} \end{array}$$

$$= \begin{array}{c|c} \alpha\overbrace{1-w\overset{*}{w}}^{-\,1/2} & \overbrace{1-w\overset{*}{w}}^{-\,1/2}w \\ \hline \alpha\overbrace{1-\overset{*}{w}w}^{-\,1/2}\overset{*}{w} & \overbrace{1-\overset{*}{w}w}^{-\,1/2} \end{array}\; \begin{array}{c|c} \overbrace{1-w\overset{*}{w}}^{-\,1/2} & -\overbrace{1-w\overset{*}{w}}^{-\,1/2}w \\ \hline -\overbrace{1-\overset{*}{w}w}^{-\,1/2}\overset{*}{w} & \overbrace{1-\overset{*}{w}w}^{-\,1/2} \end{array} = \begin{array}{c|c} \underbrace{\alpha-w\overset{*}{w}}_{}\overbrace{1-w\overset{*}{w}}^{-1} & \underbrace{1-\alpha}_{}\overbrace{1-w\overset{*}{w}}^{-1}w \\ \hline \underbrace{\alpha-1}_{}\overbrace{1-\overset{*}{w}w}^{-1}\overset{*}{w} & \underbrace{1-\iota\overset{*}{v}w}_{}\overbrace{1-\overset{*}{w}w}^{-1} \end{array}$$

$${}^{z}s_w^\alpha = \overbrace{1-w\overset{*}{w}}^{-1}\left(\underbrace{\alpha-w\overset{*}{w}}_{}z+(1-\alpha)\,w\right)\overbrace{(\alpha-1)\,\overset{*}{w}z+a-\iota\overset{*}{v}w}^{-1}\,\overbrace{1-\overset{*}{w}w}$$

$$s_w^1 = id\colon s_w^{-1} = s_w\colon s_w^0 = w \text{ cst}$$