

$$\underbrace{\rho \bowtie \Phi}_z = \rho^z z \underline{\Phi} - \widehat{\rho_z \mathbf{J}} z \Phi$$

$$\widehat{\mathbf{N}}_z \Omega = \widehat{\mathbf{F}} \widehat{\mathbf{T}} \widehat{\mathbf{z} \underline{\mathbf{J}}} - \widehat{\mathbf{T}} \widehat{\mathbf{F}} \widehat{\mathbf{z} \underline{\mathbf{J}}} + \widehat{\mathbf{F}} \widehat{\mathbf{z} \mathbf{J}} \times \widehat{\mathbf{F}} \widehat{\mathbf{z} \mathbf{J}}$$

$$\begin{aligned} \underbrace{\rho \bowtie \rho \bowtie \Phi}_z &= \underbrace{\rho \bowtie \rho^z z \underline{\Phi}} - \widehat{\rho \bowtie \rho^z z \mathbf{J}} z \Phi = \widehat{\rho^z \rho^z - \rho^z \rho^z} z \underline{\Phi} - \widehat{\rho^z \rho^z - \rho^z \rho^z} z \mathbf{J} z \Phi \\ \underbrace{\rho \bowtie \widehat{\rho \bowtie \Phi}}_z &= \rho^z \underbrace{\widehat{\rho \bowtie \Phi}}_w - \widehat{\rho^z z \mathbf{J}} \underbrace{\rho \bowtie \Phi}_z = \rho^z \underbrace{\rho^w w \underline{\Phi}} - \widehat{\rho^w w \mathbf{J}} w \Phi - \widehat{\rho^z z \mathbf{J}} \underbrace{\rho^z z \underline{\Phi}} - \widehat{\rho^z z \mathbf{J}} z \Phi \\ &= \widehat{\rho^z \rho^z} z \underline{\Phi} + \widehat{\rho^z \rho^z} z \underline{\Phi} - \widehat{\rho^z \rho^z} z \mathbf{J} z \Phi - \widehat{\rho^z \rho^z} z \mathbf{J} z \Phi - \widehat{\rho^z z \mathbf{J}} \widehat{\rho^z z \underline{\Phi}} - \widehat{\rho^z z \mathbf{J}} \widehat{\rho^z z \underline{\Phi}} + \widehat{\rho^z z \mathbf{J}} \widehat{\rho^z z \mathbf{J}} z \Phi \\ \widehat{\rho^z \rho^z} z \underline{\Omega} z \Phi &= \underbrace{\rho \bowtie \widehat{\rho \bowtie \Phi}}_z - \underbrace{\rho \bowtie \widehat{\rho \bowtie \Phi}}_z - \underbrace{\rho \bowtie \widehat{\rho \bowtie \Phi}}_z \\ &= \widehat{\rho^z \rho^z} z \underline{\Phi} + \widehat{\rho^z \rho^z} z \underline{\Phi} - \widehat{\rho^z \rho^z} z \mathbf{J} z \Phi - \widehat{\rho^z \rho^z} z \mathbf{J} z \Phi - \widehat{\rho^z z \mathbf{J}} \widehat{\rho^z z \underline{\Phi}} - \widehat{\rho^z z \mathbf{J}} \widehat{\rho^z z \underline{\Phi}} + \widehat{\rho^z z \mathbf{J}} \widehat{\rho^z z \mathbf{J}} z \Phi \\ &\quad - \widehat{\rho^z \rho^z} z \underline{\Phi} - \widehat{\rho^z \rho^z} z \underline{\Phi} + \widehat{\rho^z \rho^z} z \mathbf{J} z \Phi + \widehat{\rho^z \rho^z} z \mathbf{J} z \Phi + \widehat{\rho^z z \mathbf{J}} \widehat{\rho^z z \underline{\Phi}} + \widehat{\rho^z z \mathbf{J}} \widehat{\rho^z z \underline{\Phi}} - \widehat{\rho^z z \mathbf{J}} \widehat{\rho^z z \mathbf{J}} z \Phi \\ &\quad - \widehat{\rho^z \rho^z - \rho^z \rho^z} z \underline{\Phi} + \widehat{\rho^z \rho^z - \rho^z \rho^z} z \mathbf{J} z \Phi = \widehat{\rho^z \rho^z} z \mathbf{J} - \widehat{\rho^z \rho^z} z \mathbf{J} z \Phi + \widehat{\rho^z z \mathbf{J}} \widehat{\rho^z z \mathbf{J}} z \Phi - \widehat{\rho^z z \mathbf{J}} \widehat{\rho^z z \mathbf{J}} z \Phi \end{aligned}$$

$$\widehat{\mathbf{N}}_z \Omega = \widehat{\mathbf{F}} \widehat{\mathbf{z} \underline{\mathbf{E}}} \times \widehat{\mathbf{F}} \widehat{\mathbf{z} \underline{\mathbf{E}}}$$

$$\begin{aligned} \widehat{\mathbf{N}}_z \Omega &= \widehat{\mathbf{F}} \widehat{\mathbf{T}} \widehat{\mathbf{z} \underline{\mathbf{J}}} - \widehat{\mathbf{T}} \widehat{\mathbf{F}} \widehat{\mathbf{z} \underline{\mathbf{J}}} + \widehat{\mathbf{F}} \widehat{\mathbf{z} \mathbf{J}} \times \widehat{\mathbf{F}} \widehat{\mathbf{z} \mathbf{J}} \\ &= \widehat{\mathbf{F}} \widehat{\mathbf{F}} \widehat{\mathbf{z} \underline{\mathbf{E}}} - \widehat{\mathbf{F}} \widehat{\mathbf{F}} \widehat{\mathbf{z} \underline{\mathbf{E}}} + \widehat{\mathbf{F}} \widehat{\mathbf{F}} \widehat{\mathbf{z} \underline{\mathbf{E}}} \times \widehat{\mathbf{F}} \widehat{\mathbf{F}} \widehat{\mathbf{z} \underline{\mathbf{E}}} = \underbrace{\widehat{\mathbf{F}} \widehat{\mathbf{F}} \widehat{\mathbf{z} \underline{\mathbf{E}}} - \widehat{\mathbf{F}} \widehat{\mathbf{F}} \widehat{\mathbf{z} \underline{\mathbf{E}}}}_{=0} + \widehat{\mathbf{F}} \widehat{\mathbf{F}} \widehat{\mathbf{z} \underline{\mathbf{E}}} \times \widehat{\mathbf{F}} \widehat{\mathbf{F}} \widehat{\mathbf{z} \underline{\mathbf{E}}} \end{aligned}$$

$${}_zE \times \underbrace{\nabla {}_z\Omega}_{} = 0$$

$$\begin{aligned} \uparrow_{z\underline{E}} E &= \widehat{\uparrow_{z\underline{E}}} zE + zE \widehat{\uparrow_{z\underline{E}}} \Rightarrow zE \widehat{\uparrow_{z\underline{E}}} = \uparrow_{z\underline{E}} - \widehat{\uparrow_{z\underline{E}}} zE \\ zE \widehat{\uparrow_{z\underline{E}}} \widehat{\uparrow_{z\underline{E}}} &= \underbrace{\widehat{\uparrow_{z\underline{E}}} - \widehat{\uparrow_{z\underline{E}}} zE}_{\widehat{\uparrow_{z\underline{E}}} zE} \widehat{\uparrow_{z\underline{E}}} = \widehat{\uparrow_{z\underline{E}}} \widehat{\uparrow_{z\underline{E}}} - \widehat{\uparrow_{z\underline{E}}} zE \widehat{\uparrow_{z\underline{E}}} \\ &= \widehat{\uparrow_{z\underline{E}}} \widehat{\uparrow_{z\underline{E}}} - \widehat{\uparrow_{z\underline{E}}} \widehat{\uparrow_{z\underline{E}}} - \widehat{\uparrow_{z\underline{E}}} zE = \widehat{\uparrow_{z\underline{E}}} \widehat{\uparrow_{z\underline{E}}} zE \end{aligned}$$