

$$\mathbb{J} \xleftarrow[\text{lin}]{} \mathbb{J} \ni \mathbb{J}$$

$$\min \lceil \overset{k+1}{\mathbf{x}} \rfloor = 0 \Rightarrow \lceil \overset{0}{\mathbf{x}} \rfloor \cdots \lceil \overset{k}{\mathbf{x}} \rfloor \text{ free } \mathbb{K}$$

$$\nexists \bigvee_{\overbrace{0\alpha \cdots k\alpha} \neq 0} \underline{\lceil \mathbf{x} \rfloor}_0 \alpha + \cdots + \underline{\lceil \mathbf{x} \rfloor}_k \alpha = 0 \Rightarrow \bigvee_{0 \leq j \leq k} 0\alpha = \cdots = {}_{j-1}\alpha = 0 \neq {}_j\alpha \Rightarrow \underline{\lceil \mathbf{x} \rfloor}_j \alpha + \cdots + \underline{\lceil \mathbf{x} \rfloor}_k \alpha = 0$$

$$\Rightarrow 0 = \lceil \overset{k-j}{\mathbf{x}} \rceil \overbrace{\underline{\lceil \mathbf{x} \rfloor}_j \alpha + \cdots + \underline{\lceil \mathbf{x} \rfloor}_k \alpha}^{\neq 0} = \underline{\lceil \mathbf{x} \rfloor}_j \alpha + \underline{\lceil \mathbf{x} \rfloor}_{j+1} \alpha + \cdots + \underline{\lceil \mathbf{x} \rfloor}_{k-j} \alpha = \underline{\lceil \mathbf{x} \rfloor}_j \neq 0 \nexists$$

$$<\lceil \overset{\mathbb{N}}{\mathbf{x}} \rfloor> \xleftarrow[\text{lin}]{} <\lceil \overset{\mathbb{N}}{\mathbf{x}} \rfloor>$$

$$F \underline{\lceil \mathbf{x} \rfloor} = \lambda \underline{\lceil \mathbf{x} \rfloor} + \lceil \overset{<k}{\mathbf{x}} \rfloor \Rightarrow \text{tr } F = \lambda \underline{k+1}$$

$$\begin{array}{c} \mathbb{T} \sqsubset \mathcal{A} \\ \text{ideal} \\ \mathbb{T} \xrightarrow[\text{lin}]{} \mathbb{K} \Rightarrow \mathcal{A} \times \mathbb{T} \subset \mathcal{A} \times \mathbb{T} \end{array}$$

$$\begin{array}{c} \mathbb{T} \\ \downarrow \\ \mathcal{A} \in \mathbb{T} \\ \mathcal{A} \in \mathcal{A} \end{array}$$

$$\mathcal{A} \times \underline{\mathcal{A}}^k \in \underline{\mathcal{A}} \underline{\mathcal{A}}^k + \langle \mathcal{A}^{< k} \rangle >$$

$$\begin{aligned} k=0: \quad \mathcal{A} \times \mathbb{1} &= \underline{\mathcal{A}} \underline{\mathbb{1}} + 0 \\ 0 \leq k \mapsto k+1: \quad \mathcal{A} \times \underline{\mathcal{A}}^{k+1} - \underline{\mathcal{A}} \underline{\mathcal{A}}^{k+1} &= \mathcal{A} \times \overbrace{\mathcal{A} \times \underline{\mathcal{A}}^k} - \underline{\mathcal{A}} \underline{\mathcal{A}}^{k+1} \\ &\in \underline{\mathcal{A}} \underline{\mathcal{A}}^k + \langle \mathcal{A}^{< k} \rangle \\ &= \mathcal{A} \times \underbrace{\mathcal{A} \times \underline{\mathcal{A}}^k}_{\in \mathcal{A}} - \underline{\mathcal{A}} \underline{\mathcal{A}}^{k+1} + \underbrace{\mathcal{A} \times \mathcal{A} \times \underline{\mathcal{A}}^k}_{\in \mathcal{A}} \\ &\in \mathcal{A} \times \langle \mathcal{A}^{< k} \rangle + \underline{\mathcal{A}} \underline{\mathcal{A}}^k + \langle \mathcal{A}^{< k} \rangle > \subset \langle \mathcal{A}^{< k} \rangle > \end{aligned}$$

$$\langle \mathcal{A}^{\mathbb{N}} \rangle < \mathcal{A} \times \mathbb{1} >$$

$$\text{tr}_{\langle \mathcal{A}^{\mathbb{N}} \rangle} \mathcal{A} \times \mathbb{1} = \underline{\mathcal{A}} \dim \langle \mathcal{A}^{\mathbb{N}} \rangle$$

$$\mathcal{A} \times \mathbb{1} = 0$$

$$\begin{aligned} \langle \mathcal{A}^{\mathbb{N}} \rangle &< \mathcal{A} \times \mathbb{1} > \\ \dim \langle \mathcal{A}^{\mathbb{N}} \rangle & \mathcal{A} \times \mathbb{1} = \text{tr}_{\langle \mathcal{A}^{\mathbb{N}} \rangle} \underline{\mathcal{A} \times \mathbb{1}} = \text{tr}_{\langle \mathcal{A}^{\mathbb{N}} \rangle} \underline{\mathcal{A} \times \mathcal{A} \times \mathbb{1}} = 0 \end{aligned}$$

$$\mathcal{A} \times \underline{\mathcal{A}} \underline{\mathbb{1}} = \mathcal{A} \times \mathcal{A} \times \mathbb{1} + \underline{\mathcal{A} \times \mathcal{A} \times \mathbb{1}} = \underline{\mathcal{A}} \underline{\mathcal{A}} \underline{\mathbb{1}} = \mathcal{A} \times \mathbb{1} \Rightarrow \mathcal{A} \times \mathbb{1} \in \mathbb{T}$$

$$\mathfrak{b} \times_{\text{solv}} \mathbb{J} \Rightarrow \bigvee_{\substack{\mathfrak{b} \xrightarrow{\text{lin}} \mathbb{K}}} \mathfrak{b} \mathbb{J} \neq 0$$

$$\text{Ind } d = \dim \mathfrak{b} \geqslant 0$$

$$0 \leqslant d - 1 \mapsto d: \dim \mathfrak{b} = \dim \mathfrak{b}_{\text{solv}} \Rightarrow \mathfrak{b} \times \mathfrak{b} \subseteq \mathfrak{b} \Rightarrow 0 \neq \mathfrak{b} \cap \mathfrak{b} \times \mathfrak{b} \text{ abel}$$

$$\mathfrak{b} \xrightarrow[\text{lin}]{} \text{codim } 1 \quad \mathfrak{b} \cap \mathfrak{b} \times \mathfrak{b} \Rightarrow \mathfrak{b} = \pi^{-1} \mathfrak{b} \xrightarrow[\text{ideal}]{} \text{codim } 1 \quad \mathfrak{b}$$

$$\Rightarrow \bigvee_{\substack{\mathfrak{b} \xrightarrow[\text{lin}]{} \mathbb{K}}} \mathfrak{b} \mathbb{J} \neq 0$$

$$\mathfrak{b} \in \mathfrak{b} \cap \mathfrak{b} \times \mathfrak{b} \subseteq \bigvee_{\substack{\mathfrak{b} \xrightarrow[\text{alg abg}]{} \mathbb{J}}} \mathfrak{b} \mathbb{J} \Rightarrow \bigvee_{\substack{\mathfrak{b} \xrightarrow[\text{alg abg}]{} \mathbb{J}}} 0 \neq \mathbb{J} \in \bigvee_{\substack{\mathfrak{b} \xrightarrow[\text{alg abg}]{} \mathbb{J}}} \mathfrak{b} \mathbb{J}$$

$$\mathfrak{b} \times \mathbb{J} = \mathbb{J} \lambda$$

$$\mathfrak{b} \mathbb{K} + \mathfrak{b} = \mathfrak{b} \xrightarrow[\text{lin}]{} \mathbb{K}$$

$$\underline{b}\alpha + \underline{b} \mathfrak{b} = \lambda \alpha + \mathfrak{b} \mathfrak{b}$$

$$\underline{b}\alpha + \underline{b} \times \mathbb{J} = \underline{b} \times \mathbb{J} \alpha + \underline{b} \times \mathbb{J} = \underline{\mathbb{J}} \lambda \alpha + \underline{\mathfrak{b}} \mathfrak{b} \mathbb{J} = \mathbb{J} \lambda \alpha + \underline{\mathfrak{b}} \mathfrak{b} \mathbb{J} = \mathbb{J} \underline{\mathfrak{b}} \alpha + \underline{\mathfrak{b}} \mathfrak{b} \mathbb{J}$$