$$\mathbb{C}^{1|1}\!\!\diagup\!\!\!\diagdown_{\infty}\mathbb{C} \xleftarrow[\text{Ber}]{\overset{*}{\mathcal{T}}\mathcal{T}} \mathbb{C}^{1|1}\!\!\diagup\!\!\!\diagdown_{\infty}\mathbb{C}$$

$$\mathcal{B} = \left(\begin{array}{cc|cc} \varepsilon\varkappa\overset{2}{\overline{1-\delta}}\nu & \varepsilon\varkappa\underbrace{1-\overline{\delta}^{2}} & 0 & 0 \\ 0 & \overset{2}{\overline{1-\delta}}\nu & 0 & 0 \\ \hline 0 & 0 & \varepsilon\overset{2}{\overline{1-\delta}}\bar{\nu} & 0 \\ 0 & 0 & 0 & \varkappa\overset{2}{\overline{1-\delta}} \end{array}\right) \boldsymbol{\varkappa} \mathcal{B}^{\nu} = \varepsilon\varkappa\overset{2}{\overline{1-\delta}}\,\nu \left(\begin{array}{cc|cc} 1 & \dfrac{1-\overline{\delta}^{2}}{\overset{2}{\overline{1-\delta}}\nu} & 0 & 0 \\ 0 & \varepsilon\varkappa & 0 & 0 \\ \hline 0 & 0 & \nu/\varkappa & 0 \\ 0 & 0 & 0 & \varepsilon/\nu \end{array}\right) \boldsymbol{\varkappa} \mathcal{B}^{\nu}$$

$$\mathcal{B}^{\nu} f = B^{\nu\,00}\mathbb{J} - \frac{1}{\nu} B^{\nu\,11}\mathbb{J} + \zeta B^{\nu\,10}\mathbb{J} + \bar{\zeta}\, B^{\nu\,01}\mathbb{J} + \zeta\bar{\zeta}\, B^{\nu\,11}\mathbb{J}$$

$$\text{LHS} = \sigma\left(\mathcal{T}_{f}^{\nu}\right) = \sigma \left(\begin{array}{c|c} T^{\nu}_{00_{\mathbb{J}}-11_{\mathbb{J}}/\nu} & T^{\nu}_{01_{\mathbb{J}}/\nu} \\ \hline T^{\nu}_{10_{\mathbb{J}}} & T^{\nu}_{00_{\mathbb{J}}} \end{array}\right)$$

$$= \sigma T^{\nu}_{00_{\mathbb{J}}-11_{\mathbb{J}}/\nu} + \zeta\sigma T^{\nu}_{10_{\mathbb{J}}} + \bar{\zeta}\,\nu\sigma T^{\nu}_{01_{\mathbb{J}}/\nu} + \zeta\bar{\zeta}\,\nu\,\underbrace{\sigma T^{\nu}_{00_{\mathbb{J}}} - \sigma\overbrace{T^{\nu}_{00_{\mathbb{J}}-11_{\mathbb{J}}/\nu}}}} = \text{RHS}$$

$$\mathcal{B}^{\nu} = \exp\frac{1}{\nu}\left(\partial_z\bar{\partial}_z + \partial_\zeta\bar{\partial}_\zeta\right)$$

$$\exp\frac{1}{\nu}\left(\partial_z\bar{\partial}_z + \partial_\zeta\bar{\partial}_\zeta\right) f = \exp\frac{1}{\nu}\partial_z\,\bar{\partial}_z \exp\frac{1}{\nu}\partial_\zeta\,\bar{\partial}_\zeta f$$

$$= \exp\frac{1}{\nu}\partial_z\,\bar{\partial}_z\left(1 + \frac{1}{\nu}\partial_\zeta\bar{\partial}_\zeta\right)\overbrace{{}^{00}\mathbb{J} + \zeta^{10}\mathbb{J} + \bar{\zeta}^{01}\mathbb{J} + \zeta\bar{\zeta}^{11}\mathbb{J}} = \exp\frac{1}{\nu}\partial_z\,\bar{\partial}_z\overbrace{{}^{00}\mathbb{J} + \zeta^{10}\mathbb{J} + \bar{\zeta}^{01}\mathbb{J} + \zeta\bar{\zeta}^{11}\mathbb{J} - \frac{1}{\nu}{}^{11}\mathbb{J}}$$

$$= \exp\frac{1}{\nu}\partial_z\,\bar{\partial}_z\overbrace{{}^{00}\mathbb{J} - \frac{1}{\nu}{}^{11}\mathbb{J}} + \zeta\exp\frac{1}{\nu}\partial_z\,\bar{\partial}_z\,{}^{10}\mathbb{J} + \bar{\zeta}\,\exp\frac{1}{\nu}\partial_z\,\bar{\partial}_z\,{}^{01}\mathbb{J} + \zeta\bar{\zeta}\,\exp\frac{1}{\nu}\partial_z\,\bar{\partial}_z\,{}^{11}\mathbb{J}$$

$$= B^{\nu}\overbrace{{}^{00}\mathbb{J} - \frac{1}{\nu}{}^{11}\mathbb{J}} + \zeta B^{\nu\,10}\mathbb{J} + \bar{\zeta}\,B^{\nu\,01}\mathbb{J} + \zeta\bar{\zeta}\,B^{\nu\,11}\mathbb{J} = \mathcal{B}^{\nu} f$$

$$\text{allg}\,\underbrace{\overbrace{{}^{00}\mathbb{J} + \zeta^{10}\mathbb{J} + \bar{\zeta}^{01}\mathbb{J} + \bar{\zeta}\zeta^{11}\mathbb{J}}} = \underbrace{\left(\begin{array}{c|c} a\overbrace{{}^{00}\mathbb{J}} + \alpha\overbrace{{}^{11}\mathbb{J}} & b\overbrace{{}^{01}\mathbb{J}} \\ \hline c\overbrace{{}^{10}\mathbb{J}} & d\overbrace{{}^{00}\mathbb{J}} + \delta\overbrace{{}^{11}\mathbb{J}} \end{array}\right)}$$

$$= \varepsilon\varkappa\overbrace{\bar{\alpha}\underbrace{a\overbrace{{}^{00}\mathbb{J}} + \alpha\overbrace{{}^{11}\mathbb{J}}} - \bar{\delta}\underbrace{d\overbrace{{}^{00}\mathbb{J}} + \delta\overbrace{{}^{11}\mathbb{J}}} + \varepsilon\zeta\,\bar{c}\,\underbrace{c\overbrace{{}^{10}\mathbb{J}}} + \varkappa\bar{\zeta}\,\bar{b}\,\underbrace{b\overbrace{{}^{01}\mathbb{J}}} + \bar{\zeta}\,\zeta\overbrace{\bar{a}\underbrace{a\overbrace{{}^{00}\mathbb{J}} + \alpha\overbrace{{}^{11}\mathbb{J}}} - \bar{d}\underbrace{d\overbrace{{}^{00}\mathbb{J}} + \delta\overbrace{{}^{11}\mathbb{J}}}}$$

$$= \varepsilon\varkappa\underbrace{\bar{\alpha}a - \bar{\delta}d}\underbrace{\overset{00}{\Downarrow}} + \varepsilon\varkappa\underbrace{\overset{2}{\alpha} - \overset{2}{\delta}}\underbrace{\overset{11}{\Downarrow}} + \varepsilon\zeta\overset{2}{c}\underbrace{\overset{10}{\Downarrow}} + \varkappa\bar{\zeta}\overset{2}{b}\underbrace{\overset{01}{\Downarrow}} + \bar{\zeta}\zeta\overbrace{\underbrace{\overset{2}{a} - \overset{2}{d}}\underbrace{\overset{00}{\Downarrow}} + \underbrace{\bar{a}\alpha - \bar{d}\delta}\underbrace{\overset{11}{\Downarrow}}}$$

$$\mathrm{geo}\delta\ \mathrm{calc}\ \overbrace{\underbrace{\overset{00}{\Downarrow} + \zeta\overset{10}{\Downarrow} + \bar{\zeta}\overset{01}{\Downarrow} + \bar{\zeta}\zeta\overset{11}{\Downarrow}}}$$

$$= \varepsilon\varkappa\underbrace{(1-\delta)\,\nu - \bar{\delta}\,(1-\delta)\,\nu}\underbrace{\overset{00}{\Downarrow}} + \varepsilon\varkappa\underbrace{1 - \overset{2}{\delta}}\underbrace{\overset{11}{\Downarrow}} + \varepsilon\zeta\overset{2}{1-\delta}\,\overset{2}{\nu}\underbrace{\overset{10}{\Downarrow}} + \varkappa\bar{\zeta}\overset{2}{1-\delta}\,\overset{2}{\nu}\underbrace{\overset{01}{\Downarrow}}$$

$$+\bar{\zeta}\zeta\overbrace{\underbrace{\overset{2}{(1-\delta)\,\nu} - \overset{2}{(1-\delta)\,\nu}}_{=0}\underbrace{\overset{00}{\Downarrow}} + \underbrace{\left(1-\bar{\delta}\right)\nu - \left(1-\bar{\delta}\right)\nu\delta}\underbrace{\overset{11}{\Downarrow}}}$$

$$= \varepsilon\varkappa\underbrace{1 - \overset{2}{\delta}}\nu\underbrace{\overset{00}{\Downarrow}} + \varepsilon\varkappa\underbrace{1 - \overset{2}{\delta}}\underbrace{\overset{11}{\Downarrow}} + \varepsilon\zeta\overset{2}{1-\delta}\,\overset{2}{\nu}\underbrace{\overset{10}{\Downarrow}} + \varkappa\bar{\zeta}\overset{2}{1-\delta}\,\overset{2}{\nu}\underbrace{\overset{01}{\Downarrow}} + \bar{\zeta}\zeta\underbrace{1 - \overset{2}{\delta}}\nu\underbrace{\overset{11}{\Downarrow}}$$

$$\hat{\Delta} = \partial_z\,\bar{\partial}_z + \partial_\zeta\,\bar{\partial}_\zeta = \left(\begin{array}{cc|cc} \Delta & I & 0 & 0 \\ 0 & \Delta & 0 & 0 \\ \hline 0 & 0 & \Delta & 0 \\ 0 & 0 & 0 & \Delta \end{array}\right)$$

$$\exp\left(\frac{1-\overset{2}{\delta}}{\overset{2}{1-\delta}\,\nu}\hat{\Delta}\right) = \left(\begin{array}{ccc|cc} 1 & \dfrac{1-\overset{2}{\delta}}{\overset{2}{1-\delta}\,\nu} & & 0 & 0 \\ & & & 0 & 0 \\ 0 & 1 & & & \\ \hline & 0\ \ 0 & & 1 & 0 \\ & 0\ \ 0 & & 0 & 1 \end{array}\right) \boldsymbol{\times}\exp\left(\frac{1-\overset{2}{\delta}}{\overset{2}{1-\delta}\,\nu}\Delta\right)$$