$$\omega = d\,q^i \times d\,{}_i p$$

$$(u^{\cdot}{}_{.}u)\,\omega\,(v^{\cdot}{}_{.}v) = \begin{bmatrix} u^{\cdot} & {}_{.}u \end{bmatrix} \begin{array}{c|c} 0 & 1 \\ \hline -1 & 0 \end{array} \,\underline{\times}\, \begin{bmatrix} v^{\cdot} \\ {}_{.}v \end{bmatrix}$$

$$(u^{\cdot}{}_{.}u)\,(v^{\cdot}{}_{.}v)\,\omega = \underbrace{u^{\cdot}{}_{.}u \bowtie v^{\cdot}{}_{.}v}\,\underbrace{dq^i \bowtie d\,{}_i p} - \underbrace{u^{\cdot}{}_{.}u \bowtie v^{\cdot}{}_{.}v}\,\underbrace{d\,{}_i p \bowtie dq^i}$$

$$= \underbrace{u^{\cdot}{}_{.}u}\,\underbrace{dq^i}\,\underbrace{v^{\cdot}{}_{.}v}\,\underbrace{d\,{}_i p} - \underbrace{u^{\cdot}{}_{.}u}\,\underbrace{d\,{}_i p}\,\underbrace{v^{\cdot}{}_{.}v}\,\underbrace{dq^i} = u^i\,{}_i v - {}_i u\, v^i = \begin{bmatrix} u^{\cdot} & {}_{.}u \end{bmatrix} \begin{array}{c|c} 0 & 1 \\ \hline -1 & 0 \end{array} \,\underline{\times}\, \begin{bmatrix} v^{\cdot} \\ {}_{.}v \end{bmatrix}$$

$$\overset{\sharp}{\mathsf{L}} \times \mathsf{L} \overset{2}{\underset{\infty}{\triangleleft}} \mathbb{C} \ni {}_{x:\xi}\mathsf{J}$$

$$\mathsf{b}\,\xi\,\utilde{\mathsf{J}} = \mathsf{b}|d\daleth$$

$$\utilde{\mathsf{J}} = \begin{bmatrix} -\dfrac{\partial \daleth}{\partial\,{}_i p} & \dfrac{\partial \daleth}{\partial q^i} \end{bmatrix} \begin{bmatrix} \dfrac{\partial}{\partial q^i} \\ \dfrac{\partial}{\partial\,{}_i p} \end{bmatrix} = \frac{\partial \daleth}{\partial q^i}\frac{\partial}{\partial\,{}_i p} - \frac{\partial \daleth}{\partial\,{}_i p}\frac{\partial}{\partial q^i}$$

$$\mathsf{b} = \begin{bmatrix} \mathsf{b}^i & {}_i\mathsf{b} \end{bmatrix} \begin{bmatrix} \dfrac{\partial}{\partial q^i} \\ \dfrac{\partial}{\partial\,{}_i p} \end{bmatrix} = \mathsf{b}^i \frac{\partial}{\partial\,{}_i p} - {}_i\mathsf{b} \frac{\partial}{\partial q^i}$$

$$\mathsf{b}\,\xi\,\utilde{\mathsf{J}} = \begin{bmatrix} \mathsf{b}^i & {}_i\mathsf{b} \end{bmatrix} \begin{array}{c|c} 0 & 1 \\ \hline -1 & 0 \end{array} \begin{bmatrix} \utilde{\mathsf{J}}^i \\ {}_i\utilde{\mathsf{J}} \end{bmatrix} = \mathsf{b}^i\,{}_i\utilde{\mathsf{J}} - {}_i\mathsf{b}\,\utilde{\mathsf{J}}^i$$

$$\mathsf{b}|d\daleth = \begin{bmatrix} \mathsf{b}^i & {}_i\mathsf{b} \end{bmatrix} \begin{bmatrix} \dfrac{\partial \daleth}{\partial q^i} \\ \dfrac{\partial \daleth}{\partial\,{}_i p} \end{bmatrix} = \mathsf{b}^i \frac{\partial \daleth}{\partial q^i} + {}_i\mathsf{b} \frac{\partial \daleth}{\partial\,{}_i p} \Longrightarrow \begin{cases} \dfrac{\partial \daleth}{\partial q^i} & = {}_i\utilde{\mathsf{J}} \\ \dfrac{\partial \daleth}{\partial\,{}_i p} & -\utilde{\mathsf{J}}^i \end{cases}$$

$$\mathsf{J} \times \text{ɟ} = \utilde{\mathsf{J}} \ltimes \text{ɟ} = \utilde{\mathsf{J}}|d\,\text{ɟ} = \utilde{\mathsf{J}}\,\omega\,\utilde{\text{ɟ}}$$

$$\mathsf{J} \times \text{Ɉ} = \frac{\partial \mathsf{J}}{\partial q^i}\frac{\partial \text{Ɉ}}{\partial p_i} - \frac{\partial \mathsf{J}}{\partial p_i}\frac{\partial \text{Ɉ}}{\partial q^i} = \sum_i \frac{\partial \mathsf{J}}{\partial x^i}\frac{\partial \text{Ɉ}}{\partial \xi_i} - \frac{\partial \text{Ɉ}}{\partial x^i}\frac{\partial \mathsf{J}}{\partial \xi_i}$$

$$\mathsf{J} \times \text{Ɉ} = \begin{bmatrix} -\frac{\partial \mathsf{J}}{\partial p_i} & \frac{\partial \mathsf{J}}{\partial q^i} \end{bmatrix} \begin{array}{c|c} 0 & 1 \\ \hline -1 & 0 \end{array} \times \begin{bmatrix} -\frac{\partial \text{Ɉ}}{\partial p_i} \\ \frac{\partial \text{Ɉ}}{\partial q^i} \end{bmatrix} = \frac{-\partial \mathsf{J}}{\partial p_i}\frac{\partial \text{Ɉ}}{\partial q^i} - \frac{\partial \mathsf{J}}{\partial q^i}\frac{-\partial \text{Ɉ}}{\partial p_i} = \frac{\partial \mathsf{J}}{\partial q^i}\frac{\partial \text{Ɉ}}{\partial p_i} - \frac{\partial \mathsf{J}}{\partial p_i}\frac{\partial \text{Ɉ}}{\partial q^i}$$

$$\mathsf{J} \times \text{Ɉ} = \begin{bmatrix} \frac{\partial \mathsf{J}}{\partial q^i} \\ \frac{\partial \mathsf{J}}{\partial p_i} \end{bmatrix} \times \begin{array}{c|c} 0 & 1 \\ \hline -1 & 0 \end{array} \begin{bmatrix} \frac{\partial \text{Ɉ}}{\partial q^i} \\ \frac{\partial \text{Ɉ}}{\partial p_i} \end{bmatrix}$$

$$\begin{array}{c|c} 0 & 1 \\ \hline -1 & 0 \end{array} = -\begin{array}{c|c} 0 & 1 \\ \hline -1 & 0 \end{array}^{-1}$$