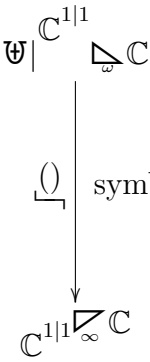

$$\overset{*}{\bar{\eta}} = \varepsilon\,\bar{\eta}$$

$$\bar{\eta}^* = \varkappa\eta$$

$$\overbrace{\bar{\eta}\eta}^{*} = \eta^*\,\bar{\eta}^* = \varepsilon\varkappa\,\bar{\eta}\,\eta$$

$$\int_{dw/\pi}^{\mathbb{C}^{1|0}} \int_{d\omega}^{\mathbb{C}^{0|1}} {}_{w|\omega}\mathbb{J}_{w|\omega} = \overline{\langle \mathbb{J} \rangle}$$ super-trace

$$\overbrace{{}^{00}\mathbb{J}+\zeta^{10}\mathbb{J}+\bar{\zeta}^{01}\mathbb{J}+\bar{\zeta}\zeta^{11}\mathbb{J}} = \frac{a\overbrace{{}^{00}\mathbb{J}}+\alpha\overbrace{{}^{11}\mathbb{J}} \;\Big|\; b\overbrace{{}^{01}\mathbb{J}}}{c\overbrace{{}^{10}\mathbb{J}} \;\Big|\; d\overbrace{{}^{00}\mathbb{J}}+\delta\overbrace{{}^{11}\mathbb{J}}}$$

$$\Rightarrow \underbrace{\begin{array}{c|c}\mathbb{1}&\mathbb{2}\\\hline\mathbb{3}&\mathbb{4}\end{array}} = \begin{cases}\bar{\alpha}\mathbb{1}-\bar{\delta}\mathbb{4}+\zeta\bar{c}\mathbb{3}+\bar{\zeta}\bar{b}\mathbb{2}+\bar{\zeta}\zeta\overbrace{\bar{a}\mathbb{1}-\bar{d}\mathbb{4}}\\ \dfrac{\bar{\alpha}\mathbb{1}-\bar{\delta}\mathbb{4}}{\varepsilon\varkappa}+\zeta\dfrac{\bar{c}\mathbb{3}}{\varepsilon}+\bar{\zeta}\dfrac{\bar{b}\mathbb{2}}{\varkappa}+\bar{\zeta}\zeta\overbrace{\bar{a}\mathbb{1}-\bar{d}\mathbb{4}}\end{cases}$$

$$\underbrace{\begin{array}{c|c}\mathbb{1}&\mathbb{2}\\\hline\mathbb{3}&\mathbb{4}\end{array}} = {}^{00}\sigma+\omega^{10}\sigma+\bar{\omega}^{01}\sigma+\bar{\omega}\omega^{11}\sigma$$

$$\int\limits_{d^2w/\pi}^{\mathbb{C}^{1|0}}\int\limits_{d^2\omega}^{\mathbb{C}^{0|1}}\overbrace{{}^{00}\mathbb{J}+\omega^{10}\mathbb{J}+\bar{\omega}^{01}\mathbb{J}+\bar{\omega}\omega^{11}\mathbb{J}}^{*}\underbrace{{}^{00}\sigma+\omega^{10}\sigma+\bar{\omega}^{01}\sigma+\bar{\omega}\omega^{11}\sigma}$$

$$=\int\limits_{d^2w/\pi}^{\mathbb{C}^{1|0}}\int\limits_{d^2\omega}^{\mathbb{C}^{0|1}}\begin{cases}\overbrace{{}^{00*}\mathbb{J}+\bar{\omega}^{10*}\mathbb{J}+\omega^{01*}\mathbb{J}+\bar{\omega}\omega^{11*}\mathbb{J}}\\ \overbrace{{}^{00*}\mathbb{J}+\varepsilon\bar{\omega}^{10*}\mathbb{J}+\varkappa\omega^{01*}\mathbb{J}+\varepsilon\varkappa\bar{\omega}\omega^{11*}\mathbb{J}}\end{cases}\underbrace{{}^{00}\sigma+\omega^{10}\sigma+\bar{\omega}^{01}\sigma+\bar{\omega}\omega^{11}\sigma}$$

$$=\int\limits_{d^2w/\pi}^{\mathbb{C}^{1|0}}\int\limits_{d^2\omega}^{\mathbb{C}^{0|1}}\begin{cases}{}^{00*}\mathbb{J}^{00}\sigma+\omega\underbrace{{}^{01*}\mathbb{J}^{00}\sigma+{}^{00*}\mathbb{J}^{10}\sigma}+\bar{\omega}\underbrace{{}^{10*}\mathbb{J}^{00}\sigma+{}^{00}\mathbb{J}^{01}\sigma}+\bar{\omega}\omega\underbrace{{}^{11*}\mathbb{J}^{00}\sigma+{}^{10*}\mathbb{J}^{10}\sigma-{}^{01*}\mathbb{J}^{01}\sigma+{}^{00*}\mathbb{J}^{11}\sigma}\\ {}^{00*}\mathbb{J}^{00}\sigma+\omega\underbrace{\varkappa^{01*}\mathbb{J}^{00}\sigma+{}^{00*}\mathbb{J}^{10}\sigma}+\bar{\omega}\underbrace{\varepsilon^{10*}\mathbb{J}^{00}\sigma+{}^{00}\mathbb{J}^{01}\sigma}+\bar{\omega}\omega\underbrace{\varepsilon\varkappa^{11*}\mathbb{J}^{00}\sigma+\varepsilon^{10*}\mathbb{J}^{10}\sigma-\varkappa^{01*}\mathbb{J}^{01}\sigma+{}^{00*}\mathbb{J}^{11}\sigma}\end{cases}$$

$$=\int\limits_{d^2w/\pi}^{\mathbb{C}^{1|0}}\begin{cases}\underbrace{{}^{11*}\mathbb{J}^{00}\sigma+{}^{10*}\mathbb{J}^{10}\sigma-{}^{01*}\mathbb{J}^{01}\sigma+{}^{00*}\mathbb{J}^{11}\sigma}\\ \underbrace{\varepsilon\varkappa^{11*}\mathbb{J}^{00}\sigma+\varepsilon^{10*}\mathbb{J}^{10}\sigma-\varkappa^{01*}\mathbb{J}^{01}\sigma+{}^{00*}\mathbb{J}^{11}\sigma}\end{cases} = \underbrace{\overset{*}{\mathbb{J}}\underbrace{\begin{array}{c|c}\mathbb{1}&\mathbb{2}\\\hline\mathbb{3}&\mathbb{4}\end{array}}} = \underbrace{\frac{\bar{a}\overbrace{{}^{00}\mathbb{J}}^{*}+\bar{\alpha}\overbrace{{}^{11}\mathbb{J}}^{*} \;\Big|\; \bar{c}\overbrace{{}^{10}\mathbb{J}}^{*}}{\bar{b}\overbrace{{}^{01}\mathbb{J}}^{*} \;\Big|\; \bar{d}\overbrace{{}^{00}\mathbb{J}}^{*}+\bar{\delta}\overbrace{{}^{11}\mathbb{J}}^{*}}\frac{\mathbb{1}\;|\;\mathbb{2}}{\mathbb{3}\;|\;\mathbb{4}}}$$

$$=\underbrace{\bar{a}\overbrace{{}^{00}\mathbb{J}}^{*}+\bar{\alpha}\overbrace{{}^{11}\mathbb{J}}^{*}}\mathbb{1}+\bar{c}\overbrace{{}^{10}\mathbb{J}}^{*}\mathbb{3}-\underbrace{\bar{b}\overbrace{{}^{01}\mathbb{J}}^{*}\mathbb{2}+\bar{d}\overbrace{{}^{00}\mathbb{J}}^{*}+\bar{\delta}\overbrace{{}^{11}\mathbb{J}}^{*}}\mathbb{4}$$

$$=\int\limits_{d^2w/\pi}^{\mathbb{C}^{1|0}}\underbrace{\bar{a}^{00*}\mathbb{J}+\bar{\alpha}^{11*}\mathbb{J}}\mathbb{1}+\bar{c}^{\,10*}\mathbb{J}\mathbb{3}-\bar{b}^{\,01*}\mathbb{J}\mathbb{2}-\underbrace{\bar{d}^{00*}\mathbb{J}+\bar{\delta}^{11*}\mathbb{J}}\mathbb{4}$$

$$=\int\limits_{d^2w/\pi}^{\mathbb{C}^{1|0}}{}^{11*}\mathbb{J}\overbrace{\bar{\alpha}\mathbb{1}-\bar{\delta}\mathbb{4}}+{}^{10*}\mathbb{J}\,\bar{c}\,\mathbb{3}-{}^{01*}\mathbb{J}\,\bar{b}\,\mathbb{2}+{}^{00*}\mathbb{J}\overbrace{\bar{a}\mathbb{1}-\bar{d}\mathbb{4}}$$

$$\Rightarrow\frac{\begin{cases}{}^{00}\sigma\\ \varepsilon\varkappa^{00}\sigma\end{cases}\;\Big|\;\begin{cases}{}^{10}\sigma\\ \varepsilon^{10}\sigma\end{cases}}{\begin{cases}{}^{01}\sigma\\ \varkappa^{01}\sigma\end{cases}\;\Big|\;{}^{11}\sigma} = \frac{\bar{\alpha}\mathbb{1}-\bar{\delta}\mathbb{4}\;\Big|\;\bar{c}\mathbb{3}}{\bar{b}\mathbb{2}\;\Big|\;\bar{a}\mathbb{1}-\bar{d}\mathbb{4}}$$

$$\underbrace{\left[\begin{array}{c|c} \llcorner & \urcorner \\ \hline \llcorner & \urcorner \end{array}\right]} = \underset{\nu}{\llcorner} + \zeta \underset{\nu}{\llcorner} + \nu\bar{\zeta} \underset{\nu}{\urcorner} + \nu\zeta\bar{\zeta} \underbrace{\urcorner - \llcorner}_{\nu} \text{ Toeplitz}$$

$$\underbrace{\left[\begin{array}{c|c} \llcorner & \urcorner \\ \hline \llcorner & \urcorner \end{array}\right]} = {}^{00}\sigma + \omega\,{}^{10}\sigma + \bar{\omega}\,{}^{01}\sigma + \omega\bar{\omega}\,{}^{11}\sigma$$

$$\int_{dw/\pi}^{\mathbb{C}^{1|0}} \int_{d\omega}^{\mathbb{C}^{0|1}} {}_{w|\omega}\mathbb{J} \underbrace{\left[\begin{array}{c|c} \llcorner & \urcorner \\ \hline \llcorner & \urcorner \end{array}\right]}_{w|\omega} = \int_{dw/\pi}^{\mathbb{C}^{1|0}} \int_{d\omega}^{\mathbb{C}^{0|1}} \overbrace{{}^{00}\mathbb{J} + \omega\,{}^{10}\mathbb{J} + \bar{\omega}\,{}^{01}\mathbb{J} + \omega\bar{\omega}\,{}^{11}\mathbb{J}} \underbrace{{}^{00}\sigma + \omega\,{}^{10}\sigma + \bar{\omega}\,{}^{01}\sigma + \omega\bar{\omega}\,{}^{11}\sigma}$$

$$= \int_{dw/\pi}^{\mathbb{C}^{1|0}} \int_{d\omega}^{\mathbb{C}^{0|1}} {}^{00}\mathbb{J}\,{}^{00}\sigma + \omega \overbrace{{}^{10}\mathbb{J}{}^{00}\sigma + {}^{00}\mathbb{J}{}^{10}\sigma} + \bar{\omega} \overbrace{{}^{01}\mathbb{J}{}^{00}\sigma + {}^{00}\mathbb{J}{}^{01}\sigma} + \omega\bar{\omega} \overbrace{{}^{11}\mathbb{J}{}^{00}\sigma + {}^{00}\mathbb{J}{}^{11}\sigma + {}^{10}\mathbb{J}{}^{01}\sigma - {}^{01}\mathbb{J}{}^{10}\sigma}$$

$$= \int_{dw/\pi}^{\mathbb{C}^{1|0}} {}^{01}\mathbb{J}\,{}^{10}\sigma - {}^{11}\mathbb{J}\,{}^{00}\sigma - {}^{00}\mathbb{J}\,{}^{11}\sigma - {}^{10}\mathbb{J}\,{}^{01}\sigma = \underbrace{\mathbb{J}\left[\begin{array}{c|c} \llcorner & \urcorner \\ \hline \llcorner & \urcorner \end{array}\right]} = \underbrace{\left[\begin{array}{c|c} \overbrace{{}^{00}\mathbb{J} - {}^{11}\mathbb{J}/\nu}^{\nu} & \overbrace{{}^{01}\mathbb{J}/\nu}^{\nu} \\ \hline \underset{\nu}{{}^{10}\mathbb{J}} & \underset{\nu}{{}^{00}\mathbb{J}} \end{array}\right] \left[\begin{array}{c|c} \llcorner & \urcorner \\ \hline \llcorner & \urcorner \end{array}\right]}$$

$$= \underbrace{\overbrace{{}^{00}\mathbb{J} - {}^{11}\mathbb{J}/\nu}^{\nu}\ \llcorner + \overbrace{{}^{01}\mathbb{J}/\nu}^{\nu}\ \urcorner} - \underbrace{\overbrace{{}^{10}\mathbb{J}}^{\nu}\ \urcorner + \overbrace{{}^{00}\mathbb{J}}^{\nu}\ \urcorner} = \int_{\nu dw/\pi}^{\mathbb{C}} \underbrace{{}^{00}\mathbb{J} - {}^{11}\mathbb{J}/\nu} \underset{\nu}{\llcorner} + \frac{{}^{01}\mathbb{J}}{\nu} \underset{\nu}{\urcorner} - {}^{10}\mathbb{J} \underset{\nu}{\urcorner} - {}^{00}\mathbb{J} \underset{\nu}{\urcorner}$$

$$= \int_{dw/\pi}^{\mathbb{C}} \underbrace{\nu\,{}^{00}\mathbb{J} - {}^{11}\mathbb{J}} \underset{\nu}{\llcorner} + {}^{01}\mathbb{J} \underset{\nu}{\urcorner} - \nu\,{}^{10}\mathbb{J} \underset{\nu}{\urcorner} - \nu\,{}^{00}\mathbb{J} \underset{\nu}{\urcorner} = \int_{dw/\pi}^{\mathbb{C}} {}^{00}\mathbb{J}\nu \underbrace{\llcorner - \urcorner}_{\nu} - {}^{10}\mathbb{J}\nu \underset{\nu}{\urcorner} + {}^{01}\mathbb{J} \underset{\nu}{\urcorner} - {}^{11}\mathbb{J} \underset{\nu}{\llcorner}$$

$$\Rightarrow \left[\begin{array}{c|c} {}^{00}\sigma & {}^{10}\sigma \\ \hline {}^{01}\sigma & {}^{11}\sigma \end{array}\right] = \left[\begin{array}{c|c} \underset{\nu}{\llcorner} & \underset{\nu}{\urcorner} \\ \hline \nu \underset{\nu}{\urcorner} & \nu \underbrace{\urcorner - \llcorner}_{\nu} \end{array}\right]$$