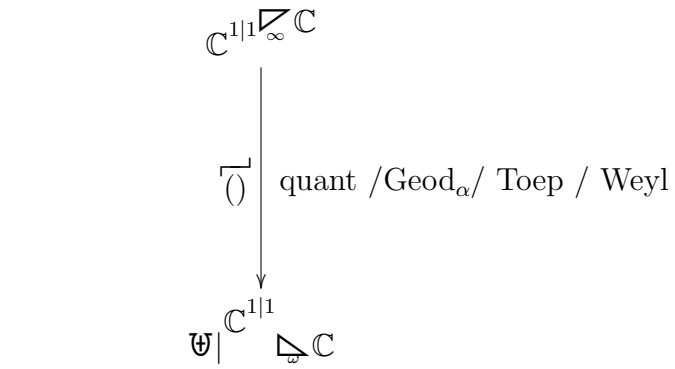

$$\text{allg}\ \overline{{}^{00}\mathbb{J} + \zeta^{10}\mathbb{J} + \bar{\zeta}^{01}\mathbb{J} + \bar{\zeta}\zeta^{11}\mathbb{J}} = \left.\begin{array}{c|c} a\,\overline{{}^{00}\mathbb{J}} + \alpha\,\overline{{}^{11}\mathbb{J}} & b\,\overline{{}^{01}\mathbb{J}} \\ \hline c\,\overline{{}^{10}\mathbb{J}} & d\,\overline{{}^{00}\mathbb{J}} + \delta\,\overline{{}^{11}\mathbb{J}} \end{array}\right.$$

$$\overbrace{{}^{00}\mathbb{J}+\zeta\,{}^{10}\mathbb{J}+\bar{\zeta}\,{}^{01}\mathbb{J}+\bar{\zeta}\zeta\,{}^{11}\mathbb{J}}=\frac{(1-\alpha)\,\nu\,\overbrace{{}^{00}\mathbb{J}}+\overbrace{{}^{11}\mathbb{J}}\;\Big|\;(1-\alpha)\,\overbrace{{}^{01}\mathbb{J}}}{(1-\alpha)\,\nu\,\overbrace{{}^{10}\mathbb{J}}\;\Big|\;(1-\alpha)\,\nu\,\overbrace{{}^{00}\mathbb{J}}+\alpha\,\overbrace{{}^{11}\mathbb{J}}}$$

$$\overbrace{{}^{z|\zeta}\mathbb{J}\,\mathbb{1}}=\int\limits_{d^2w/\pi}^{\mathbb{C}^{1|0}}\int\limits_{d^2\eta}^{\mathbb{C}^{0|1}}{}_{w|\eta}\mathbb{J}\;\overbrace{{}^{z|\zeta}w|\eta\,\mathbb{1}}=\int\limits_{d^2w/\pi}^{\mathbb{C}^{1|0}}\int\limits_{d^2\eta}^{\mathbb{C}^{0|1}}z-w|\zeta-\eta\mathfrak{e}_{w|\eta}^{(1-\alpha)\,\nu}\;{}_{w|\eta}\mathbb{J}^{\,\alpha z+(1-\alpha)\,w|\alpha\zeta+(1-\alpha)\,\eta}\mathbb{1}$$

$$=\int\limits_{d^2w/\pi}^{\mathbb{C}^{1|0}}z-w\mathfrak{e}_w^{\nu(1-\alpha)}\int\limits_{d^2\eta}^{\mathbb{C}^{0|1}}\zeta-\eta\mathfrak{e}_\eta^{(1-\alpha)\,\nu}\,\underbrace{{}^{00}_{w}\mathbb{J}+\eta\,{}^{10}_{w}\mathbb{J}+\bar{\eta}\,{}^{01}_{w}\mathbb{J}+\bar{\eta}\eta\,{}^{11}_{w}\mathbb{J}}\underbrace{{}^{\alpha z+(1-\alpha)\,w}_{0}\mathbb{1}+\overbrace{\alpha\zeta+(1-\alpha)\,\eta}\,{}^{\alpha z+(1-\alpha)\,w}_{1}\mathbb{1}}$$

$$=\int\limits_{d^2w/\pi}^{\mathbb{C}^{1|0}}z-w\mathfrak{e}_w^{\nu(1-\alpha)}\int\limits_{d^2\eta}^{\mathbb{C}^{0|1}}\underbrace{1+(1-\alpha)\,\nu\left(\zeta\bar{\eta}+\bar{\eta}\eta\right)}$$

$$\underbrace{{}^{00}_{w}\mathbb{J}+\eta\,{}^{10}_{w}\mathbb{J}+\bar{\eta}\,{}^{01}_{w}\mathbb{J}+\bar{\eta}\eta\,{}^{11}_{w}\mathbb{J}}\underbrace{{}^{\alpha z+(1-\alpha)\,w}_{0}\mathbb{1}+\alpha\zeta\,{}^{\alpha z+(1-\alpha)\,w}_{1}\mathbb{1}+(1-\alpha)\,\eta\,{}^{\alpha z+(1-\alpha)\,w}_{1}\mathbb{1}}$$

$$=\int\limits_{d^2w/\pi}^{\mathbb{C}^{1|0}}z-w\mathfrak{e}_w^{\nu(1-\alpha)}\begin{cases}(1-\alpha)\,\nu\,{}^{00}_{w}\mathbb{J}\underbrace{{}^{\alpha z+(1-\alpha)\,w}_{0}\mathbb{1}+\alpha\zeta\,{}^{\alpha z+(1-\alpha)\,w}_{1}\mathbb{1}+(1-\alpha)\,\zeta\,{}^{\alpha z+(1-\alpha)\,w}_{1}\mathbb{1}}\\+(1-\alpha)\,\nu\,{}^{10}_{w}\mathbb{J}\underbrace{{}^{\alpha z+(1-\alpha)\,w}_{0}\mathbb{1}+\alpha\zeta\,{}^{\alpha z+(1-\alpha)\,w}_{1}\mathbb{1}}\\+(1-\alpha)\,{}^{01}_{w}\mathbb{J}\,{}^{\alpha z+(1-\alpha)\,w}_{1}\mathbb{1}\\+{}^{11}_{w}\mathbb{J}\underbrace{{}^{\alpha z+(1-\alpha)\,w}_{0}\mathbb{1}+\alpha\zeta\,{}^{\alpha z+(1-\alpha)\,w}_{1}\mathbb{1}}\end{cases}$$

$$=(1-\alpha)\;\overbrace{{}^{z}\mathbb{J}_1\mathbb{1}}+\overbrace{{}^{z}\mathbb{J}_0\mathbb{1}}+\alpha\zeta\;\overbrace{{}^{z}\mathbb{J}_1\mathbb{1}}+(1-\alpha)^2\nu\zeta\;\overbrace{{}^{z}\mathbb{J}_1\mathbb{1}}$$

$$+(1-\alpha)\,\nu\zeta\;\overbrace{{}^{z}\mathbb{J}_0\mathbb{1}}+(1-\alpha)\,\nu\;\overbrace{{}^{z}\mathbb{J}_0\mathbb{1}}+(1-\alpha)\,\alpha\nu\zeta\;\overbrace{{}^{z}\mathbb{J}_1\mathbb{1}}$$

$$\overline{\mathbb{J}} \underset{\text{Toep}}{\overset{\text{glob}}{=}} \left[\begin{array}{c|c} \nu\, \overline{{}^{00}\mathbb{J}} + \overline{{}^{11}\mathbb{J}} & \overline{{}^{01}\mathbb{J}} \\ \hline \nu\, \overline{{}^{10}\mathbb{J}} & \nu\, \overline{{}^{00}\mathbb{J}} \end{array}\right]$$

$$\overbrace{\overline{\mathbb{J}}\,\daleth}^{z|\zeta} = \int\limits_{d^2 w/\pi}^{\mathbb{C}^{1|0}} \int\limits_{d^2\eta}^{\mathbb{C}^{0|1}} {}_{w|\eta}\mathbb{J}\ \overbrace{\overline{w|\eta}\,\daleth}^{z|\zeta} = \int\limits_{d^2 w/\pi}^{\mathbb{C}^{1|0}} \int\limits_{d^2\eta}^{\mathbb{C}^{0|1}} {}^{z-w|\zeta-\eta}\mathfrak{e}^{\nu}_{w|\eta}\ {}_{w|\eta}\mathbb{J}\ {}^{w|\eta}\daleth$$

$$= \int\limits_{d^2 w/\pi}^{\mathbb{C}^{1|0}} {}^{z-w}\mathfrak{e}^{\nu}_{w} \int\limits_{d^2\eta}^{\mathbb{C}^{0|1}} {}^{\zeta-\eta}\mathfrak{e}^{\nu}_{\eta} \underbrace{{}^{00}_{w}\mathbb{J} + \eta\, {}^{10}_{w}\mathbb{J} + \bar{\eta}\, {}^{01}_{w}\mathbb{J} + \bar{\eta}\eta\, {}^{11}_{w}\mathbb{J}} \underbrace{{}^{w}_{0}\daleth + \eta\, {}^{w}_{1}\daleth}$$

$$= \int\limits_{d^2 w/\pi}^{\mathbb{C}^{1|0}} {}^{z-w}\mathfrak{e}^{\nu}_{w} \int\limits_{d^2\eta}^{\mathbb{C}^{0|1}} \underbrace{1 + \nu\zeta\bar{\eta} + \nu\bar{\eta}\eta} \underbrace{{}^{00}_{w}\mathbb{J} + \eta\, {}^{10}_{w}\mathbb{J} + \bar{\eta}\, {}^{01}_{w}\mathbb{J} + \bar{\eta}\eta\, {}^{11}_{w}\mathbb{J}} \underbrace{{}^{w}_{0}\daleth + \eta\, {}^{w}_{1}\daleth}$$

$$= \int\limits_{d^2 w/\pi}^{\mathbb{C}^{1|0}} {}^{z-w}\mathfrak{e}^{\nu}_{w}\ {}^{01}_{w}\mathbb{J}\ {}^{w}_{1}\daleth + {}^{11}_{w}\mathbb{J}\ {}^{w}_{0}\daleth + \nu\zeta \underbrace{{}^{00}_{w}\mathbb{J}^{w}_{1}\daleth + {}^{10}_{w}\mathbb{J}^{w}_{0}\daleth} + \nu\, {}^{00}_{w}\mathbb{J}\ {}^{w}_{0}\daleth$$

$$= \overbrace{\overline{{}^{01}\mathbb{J}}_{1}\daleth}^{z} + \overbrace{\overline{{}^{11}\mathbb{J}}_{0}\daleth}^{z} + \nu\zeta \underbrace{\overbrace{\overline{{}^{00}\mathbb{J}}_{1}\daleth}^{z} + \overbrace{\overline{{}^{10}\mathbb{J}}_{0}\daleth}^{z}} + \nu\, \overbrace{\overline{{}^{00}\mathbb{J}}_{0}\daleth}^{z}$$

$$\Leftarrow \overbrace{\overline{\mathsf{J}}\,\daleth}^{z} = \int\limits_{\nu\, d^2 w/\pi}^{\mathbb{C}} {}^{z-w}\mathfrak{e}^{\nu}_{w}\ {}_{w}\mathsf{J}\ {}^{w}\daleth$$

$$\overbrace{\mathbb{J}\daleth}^{w|\omega} = \overbrace{{}^{00}\mathbb{J} + \omega\, {}^{10}\mathbb{J} + \bar{\omega}\, {}^{01}\mathbb{J} + \omega\bar{\omega}\, {}^{11}\mathbb{J}}\ \underbrace{{}_{0}\daleth + \omega\, {}_{1}\daleth}$$

$$= {}^{00}\mathbb{J}\,{}_{0}\daleth + \omega\, \overbrace{{}^{00}\mathbb{J}_{1}\daleth + {}^{10}\mathbb{J}_{0}\daleth} + \bar{\omega}\ {}^{01}\mathbb{J}\,{}_{0}\daleth + \omega\bar{\omega}\, \overbrace{{}^{11}\mathbb{J}_{0}\daleth - {}^{01}\mathbb{J}_{1}\daleth}$$

$$\Longrightarrow \overbrace{\overline{\mathbb{J}}\,\daleth}^{z|\zeta} = \overbrace{\mathcal{P}_{\nu}(\mathbb{J}\daleth)}^{z|\zeta} = \overbrace{P_{\nu}\ {}^{00}\mathbb{J}_{0}\daleth}^{z} + \zeta\ \overbrace{P_{\nu} \underbrace{{}^{00}\mathbb{J}_{1}\daleth + {}^{10}\mathbb{J}_{0}\daleth}}^{z} - \frac{1}{\nu} \overbrace{P_{\nu} \underbrace{{}^{11}\mathbb{J}_{0}\daleth - {}^{01}\mathbb{J}_{1}\daleth}}^{z}$$

$$= \overbrace{\overline{{}^{00}\mathbb{J}}_{0}\daleth}^{z} - \frac{1}{\nu} \overbrace{\overline{{}^{11}\mathbb{J}}_{0}\daleth}^{z} + \frac{1}{\nu} \overbrace{\overline{{}^{01}\mathbb{J}}_{1}\daleth}^{z} + \zeta\, \overbrace{\overline{{}^{10}\mathbb{J}}_{0}\daleth}^{z} + \zeta\, \overbrace{\overline{{}^{00}\mathbb{J}}_{1}\daleth}^{z}$$

$$\overbrace{{}^{00}\mathbb{J}+\zeta^{10}\mathbb{J}+\bar{\zeta}^{01}\mathbb{J}+\bar{\zeta}\zeta^{11}\mathbb{J}} \underset{\text{Weyl}}{\overset{\text{glob}}{=}} \left(\begin{array}{c|c} 2\nu\overbrace{{}^{00}\mathbb{J}}+\overbrace{{}^{11}\mathbb{J}} & 2\overbrace{{}^{01}\mathbb{J}} \\ \hline 2\nu\overbrace{{}^{10}\mathbb{J}} & 2\nu\overbrace{{}^{00}\mathbb{J}}-\overbrace{{}^{11}\mathbb{J}} \end{array}\right)$$

$$\overbrace{{}^{z|\zeta}\mathbb{J}\,\mathbb{1}} = \int\limits_{d^2w/\pi}^{\mathbb{C}^{1|0}}\int\limits_{d^2\eta}^{\mathbb{C}^{0|1}} {}_{w|\eta}\mathbb{J}\;\overbrace{{}^{z|\zeta}w|\eta\,\mathbb{1}} = \int\limits_{d^2w/\pi}^{\mathbb{C}^{1|0}}\int\limits_{d^2\eta}^{\mathbb{C}^{0|1}} z-w|\zeta-\omega\,\mathfrak{e}^{2\nu}_{w|\eta\;w|\eta}\,\mathbb{J}^{\,2w-z\big|_\eta-\zeta}\mathbb{1}$$

$$= \int\limits_{d^2w/\pi}^{\mathbb{C}^{1|0}} z-w\,\mathfrak{e}^{2\nu}_{w}\int\limits_{d^2\eta}^{\mathbb{C}^{0|1}} \zeta-\eta\,\mathfrak{e}^{2\nu}_{\eta}\,\underbrace{{}^{00}_{w}\mathbb{J}+\eta\,{}^{10}_{w}\mathbb{J}+\bar{\eta}\,{}^{01}_{w}\mathbb{J}+\bar{\eta}\eta\,{}^{11}_{w}\mathbb{J}}\underbrace{{}^{2w-z}_{0}\mathbb{1}+\overbrace{2\eta-\zeta}{}^{2w-z}_{1}\mathbb{1}}$$

$$= \int\limits_{d^2w/\pi}^{\mathbb{C}^{1|0}} z-w\,\mathfrak{e}^{2\nu}_{w}\int\limits_{d^2\eta}^{\mathbb{C}^{0|1}} \underbrace{1+2\nu\zeta\bar{\eta}+2\nu\bar{\eta}\eta}\,\underbrace{{}^{00}_{w}\mathbb{J}+\eta\,{}^{10}_{w}\mathbb{J}+\bar{\eta}\,{}^{01}_{w}\mathbb{J}+\bar{\eta}\eta\,{}^{11}_{w}\mathbb{J}}\,\underbrace{{}^{2w-z}_{0}\mathbb{1}-\zeta\,{}^{2w-z}_{1}\mathbb{1}+2\eta\,{}^{2w-z}_{1}\mathbb{1}}$$

$$= \int\limits_{d^2w/\pi}^{\mathbb{C}^{1|0}} z-w\,\mathfrak{e}^{2\nu}_{w}\begin{cases} 2\nu\,{}^{00}_{w}\mathbb{J}\underbrace{{}^{2w-z}_{0}\mathbb{1}-\zeta\,{}^{2w-z}_{1}\mathbb{1}+2\zeta\,{}^{2w-z}_{1}\mathbb{1}} \\ 2\nu\zeta\,{}^{10}_{w}\mathbb{J}\underbrace{{}^{2w-z}_{0}\mathbb{1}-\zeta\,{}^{2w-z}_{1}\mathbb{1}} \\ 2\,{}^{01}_{w}\mathbb{J}\,{}^{2w-z}_{1}\mathbb{1} \\ {}^{11}_{w}\mathbb{J}\underbrace{{}^{2w-z}_{0}\mathbb{1}-\zeta\,{}^{2w-z}_{1}\mathbb{1}} \end{cases}$$

$$= \int\limits_{d^2w/\pi}^{\mathbb{C}^{1|0}} z-w\,\mathfrak{e}^{2\nu}_{w}\,\overbrace{\underbrace{{}^{11}_{w}\mathbb{J}+2\nu\,{}^{00}_{w}\mathbb{J}}\,{}^{2w-z}_{0}\mathbb{1}+\zeta 2\nu\,{}^{10}_{w}\mathbb{J}\,{}^{2w-z}_{0}\mathbb{1}+2\,{}^{01}_{w}\mathbb{J}\,{}^{2w-z}_{1}\mathbb{1}+\zeta\underbrace{2\nu\,{}^{00}_{w}\mathbb{J}-{}^{11}_{w}\mathbb{J}}\,{}^{2w-z}_{1}\mathbb{1}}$$

$$= \int\limits_{d^2w/\pi}^{\mathbb{C}^{1|0}} z-w\,\mathfrak{e}^{2\nu}_{w}\,\underbrace{{}^{11}_{w}\mathbb{J}+2\nu\,{}^{00}_{w}\mathbb{J}}\,{}^{2w-z}_{0}\mathbb{1}+2\int\limits_{d^2w/\pi}^{\mathbb{C}^{1|0}} z-w\,\mathfrak{e}^{2\nu}_{w}\,{}^{01}_{w}\mathbb{J}\,{}^{2w-z}_{1}\mathbb{1}$$

$$+2\nu\zeta\int\limits_{d^2w/\pi}^{\mathbb{C}^{1|0}} z-w\,\mathfrak{e}^{2\nu}_{w}\,{}^{10}_{w}\mathbb{J}\,{}^{2w-z}_{0}\mathbb{1}+\zeta\int\limits_{d^2w/\pi}^{\mathbb{C}^{1|0}} z-w\,\mathfrak{e}^{2\nu}_{w}\,\underbrace{2\nu\,{}^{00}_{w}\mathbb{J}-{}^{11}_{w}\mathbb{J}}\,{}^{2w-z}_{1}\mathbb{1}$$

$$= {}^{z}\overbrace{\underbrace{{}^{11}\mathbb{J}+2\nu\,{}^{00}\mathbb{J}}_{0}\mathbb{1}}+2\,{}^{z}\overbrace{\underbrace{{}^{01}\mathbb{J}}_{1}\mathbb{1}}+2\nu\zeta\,{}^{z}\overbrace{\underbrace{{}^{10}\mathbb{J}}_{0}\mathbb{1}}+2\nu\zeta\,{}^{z}\overbrace{\underbrace{{}^{00}\mathbb{J}-{}^{11}\mathbb{J}}_{1}\mathbb{1}}$$

$$\Longleftarrow\quad {}^{z}\overbrace{\mathbb{J}\,\mathbb{1}} = \int\limits_{d^2w/\pi}^{\mathbb{C}} z-w\,\mathfrak{e}^{2\nu}_{w\;w}\mathbb{J}^{\,2w-z}\mathbb{1}$$