

$$\Psi = \begin{bmatrix} \Psi_1 & \Psi_2 \end{bmatrix} \in \mathbb{C}_2$$

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_1 \\ \mathbf{H}_2 \end{bmatrix} \in \mathbb{C}^2$$

$$\begin{cases} \Psi_R \times \Gamma = \Psi \Gamma & \Gamma \times \Psi = \Psi^{-1} \\ \mathbf{H}_R \times \Gamma = \bar{\Gamma} \mathbf{H} & \Gamma \times \mathbf{H} = \Gamma \mathbf{H} \end{cases}$$

$$\begin{cases} \Psi_L \times \Gamma = \Psi \bar{\Gamma} & \Gamma \times \Psi = \Psi \bar{\Gamma}^{-1} \\ \mathbf{H}_L \times \Gamma = \bar{\Gamma}^{-1} \mathbf{H} & \Gamma \times \mathbf{H} = \bar{\Gamma} \mathbf{H} \end{cases}$$

$$\begin{array}{ccc}
\mathbb{C}_2 & \xrightarrow{\begin{smallmatrix} \times \Gamma \\ R \\ \times \Gamma \\ L \end{smallmatrix}} & \mathbb{C}_2 \\
\downarrow \sigma^2 \mathcal{L} & \text{unit} & \downarrow \sigma^2 \mathcal{L} \\
& &
\end{array}
\quad
\begin{array}{ccc}
\mathbb{C}^2 & \xleftarrow{\begin{smallmatrix} \Gamma \times \\ R \\ \Gamma \times \\ L \end{smallmatrix}} & \mathbb{C}^2 \\
\downarrow \sigma^2 \mathcal{L} & \text{unit} & \downarrow \sigma^2 \mathcal{L} \\
& &
\end{array}
\quad
\begin{array}{ccc}
\mathbb{C}_2 & \xleftarrow{\begin{smallmatrix} \times \Gamma \\ R \\ \times \Gamma \\ L \end{smallmatrix}} & \mathbb{C}_2 \\
\downarrow \sigma^2 \mathcal{L} & \text{unit} & \downarrow \sigma^2 \mathcal{L} \\
& &
\end{array}$$

$$\sigma^2 \overbrace{\Psi_R \times \Gamma}^T = \sigma^2 \overbrace{\Psi \Gamma}^T = \sigma^2 \Gamma \Psi = \sigma^2 \Gamma \sigma^2 \Psi = \bar{\Gamma} \sigma^2 \Psi = \sigma^2 \Psi \times_R \Gamma$$

$$\sigma^2 \overbrace{\Psi_L \times \Gamma}^T = \sigma^2 \overbrace{\Psi \bar{\Gamma}}^T = \sigma^2 \Gamma \Psi = \sigma^2 \Gamma \sigma^2 \Psi = \bar{\Gamma}^{-1} \sigma^2 \Psi = \sigma^2 \Psi \times_L \Gamma$$

$$\begin{array}{ccc}
\mathbb{C}_2 & \xrightarrow{\begin{smallmatrix} \times \lrcorner \\ R \end{smallmatrix}} & \mathbb{C}_2 \\
\downarrow \sigma^2 \sim & \text{anti-unit} & \downarrow \sigma^2 \sim \\
{}^2\mathbb{C} & \xrightarrow{\begin{smallmatrix} \times \lrcorner \\ R \end{smallmatrix}} & {}^2\mathbb{C}
\end{array}
\quad
\begin{array}{ccc}
{}^2\mathbb{C} & \xleftarrow{\begin{smallmatrix} \lrcorner \times \\ R \end{smallmatrix}} & {}^2\mathbb{C} \\
\downarrow \sigma^2 \sim & \text{anti-unit} & \downarrow \sigma^2 \sim \\
\mathbb{C}_2 & \xleftarrow{\begin{smallmatrix} \lrcorner \times \\ R \end{smallmatrix}} & \mathbb{C}_2
\end{array}$$

$$\sigma^2 \overbrace{\Psi \times \lrcorner}^R = \sigma^2 \overbrace{\Psi \lrcorner}^R = \sigma^2 \lrcorner \Psi = \sigma^2 \lrcorner \sigma^2 \sigma^2 \Psi = \lrcorner^{-1} \sigma^2 \Psi = \sigma^2 \Psi \times_R \lrcorner$$

$$\sigma^2 \overbrace{\Psi \times \lrcorner}^L = \sigma^2 \overbrace{\Psi \lrcorner}^L = \sigma^2 \lrcorner \Psi = \sigma^2 \lrcorner \sigma^2 \sigma^2 \Psi = \lrcorner^{-1} \sigma^2 \Psi = \sigma^2 \Psi \times_L \lrcorner$$