

$$\frac{a}{\mathbb{L}} \left| \begin{array}{c} \mathbb{L} \\ b \end{array} \right. = a \left(\frac{1}{2} : \frac{i}{2} : 0 \right) + b \left(\frac{1}{2} : -\frac{i}{2} : 0 \right) + (0 : 0 : \mathbb{L}) = \left(\frac{a+b}{2} : i \frac{a-b}{2} : \mathbb{L} \right) : \quad a \in \mathbb{R} \ni b : \mathbb{L} \in i\mathbb{R}^a$$

$$(\alpha : \beta : \mathbb{L}) = \frac{\alpha - i\beta}{\mathbb{L}} \left| \begin{array}{c} \mathbb{L} \\ \alpha + i\beta \end{array} \right. : \quad a \in \mathbb{C} \ni b : \quad \mathbb{L} \in \mathbb{C}^a$$

$$(a : b : \mathbb{L}) = \frac{a - ib}{-\bar{\mathbb{L}}^*} \left| \begin{array}{c} \mathbb{L} \\ a + ib \end{array} \right.$$

$$\frac{a}{-\bar{\mathbb{L}}^*} \left| \begin{array}{c} \mathbb{L} \\ b \end{array} \right. = \left(\frac{a+b}{2} : i \frac{a-b}{2} : \mathbb{L} \right)$$

$$\begin{aligned} \frac{a}{-\bar{\mathbb{L}}^*} \left| \begin{array}{c} \mathbb{L} \\ b \end{array} \right. - \frac{c^*}{-\bar{\mathbb{L}}^*} \left| \begin{array}{c} \mathbb{L} \\ d \end{array} \right. - \frac{a}{-\bar{\mathbb{L}}^*} \left| \begin{array}{c} \mathbb{L} \\ b \end{array} \right. &= \frac{a}{-\bar{\mathbb{L}}^*} \left| \begin{array}{c} \mathbb{L} \\ b \end{array} \right. - \frac{\bar{c}}{\bar{\mathbb{L}}^*} \left| \begin{array}{c} -\bar{\mathbb{L}} \\ d \end{array} \right. - \frac{a}{-\bar{\mathbb{L}}^*} \left| \begin{array}{c} \mathbb{L} \\ b \end{array} \right. = \frac{a\bar{c} + \mathbb{L}\bar{\mathbb{L}}^*}{-\bar{\mathbb{L}}^*\bar{c} + b\bar{\mathbb{L}}^*} \left| \begin{array}{c} -a\bar{\mathbb{L}} + \mathbb{L}\bar{d} \\ \bar{\mathbb{L}}^*\bar{\mathbb{L}} + bd \end{array} \right. - \frac{a}{-\bar{\mathbb{L}}^*} \left| \begin{array}{c} \mathbb{L} \\ b \end{array} \right. \\ &= \frac{(a\bar{c} + \mathbb{L}\bar{\mathbb{L}}^*)a - (-a\bar{\mathbb{L}} + \mathbb{L}\bar{d})\bar{\mathbb{L}}^*}{(-\bar{\mathbb{L}}^*\bar{c} + b\bar{\mathbb{L}}^*)a - (\bar{\mathbb{L}}^*\bar{\mathbb{L}} + bd)\bar{\mathbb{L}}^*} \left| \begin{array}{c} (a\bar{c} + \mathbb{L}\bar{\mathbb{L}}^*)\mathbb{L} + (-a\bar{\mathbb{L}} + \mathbb{L}\bar{d})b \\ (-\bar{\mathbb{L}}^*\bar{c} + b\bar{\mathbb{L}}^*)\mathbb{L} + (\bar{\mathbb{L}}^*\bar{\mathbb{L}} + bd)b \end{array} \right. \\ &= \frac{a\bar{c}a + \mathbb{L}\bar{\mathbb{L}}^*a + a\bar{\mathbb{L}}\bar{\mathbb{L}}^* - \bar{\mathbb{L}}\bar{d}\bar{\mathbb{L}}^*}{-\bar{\mathbb{L}}^*\bar{c}a + b\bar{\mathbb{L}}^*a - \bar{\mathbb{L}}^*\bar{\mathbb{L}}\bar{\mathbb{L}}^* - bd\bar{\mathbb{L}}^*} \left| \begin{array}{c} a\bar{c}\mathbb{L} + \mathbb{L}\bar{\mathbb{L}}^*\mathbb{L} - a\bar{\mathbb{L}}b + \mathbb{L}\bar{d}b \\ -\bar{\mathbb{L}}^*\bar{\mathbb{L}}\bar{c} + b\bar{\mathbb{L}}^*\mathbb{L} + \bar{\mathbb{L}}^*\bar{\mathbb{L}}b + bd\bar{b} \end{array} \right. \end{aligned}$$

$$\begin{aligned} \left(\frac{a+b}{2} : \frac{b-a}{2i} : \mathbb{L} \right) \left(\frac{c+d}{2} : \frac{\bar{d}-c}{2i} : \bar{\mathbb{L}} \right) \left(\frac{a+b}{2} : \frac{b-a}{2i} : \mathbb{L} \right) &= 2 \underbrace{\left(\frac{a+b}{2} : \frac{b-a}{2i} : \mathbb{L} \right) \left(\frac{c+d}{2} : \frac{\bar{d}-c}{2i} : \bar{\mathbb{L}} \right)}_{-} \left(\frac{a+b}{2} : \frac{b-a}{2i} : \mathbb{L} \right) \\ &- \underbrace{\left(\frac{a+b}{2} : \frac{b-a}{2i} : \mathbb{L} \right) \left(\frac{a+\bar{b}}{2} : \frac{\bar{b}-a}{2i} : \bar{\mathbb{L}} \right)}_{-} \left(\frac{c+\bar{d}}{2} : \frac{\bar{d}-c}{2i} : \bar{\mathbb{L}} \right) \\ &= 2 \left(\frac{a+b}{2} \frac{\bar{c}+\bar{d}}{2} + \frac{b-a}{2i} \frac{\bar{d}-\bar{c}}{2i} + \mathbb{L}\bar{\mathbb{L}}^* \right) \left(\frac{a+b}{2} : \frac{b-a}{2i} : \mathbb{L} \right) - \left(\frac{a+b^2}{2} + \frac{b-a^2}{2i} + \mathbb{L}\bar{\mathbb{L}}^* \right) \left(\frac{c+\bar{d}}{2} : \frac{\bar{d}-c}{2i} : \bar{\mathbb{L}} \right) \\ &= 2 \left(\frac{a+b}{2} \frac{\bar{c}+\bar{d}}{2} + \frac{b-a}{2i} \frac{\bar{d}-\bar{c}}{2i} + \mathbb{L}\bar{\mathbb{L}}^* \right) \frac{\frac{a+b}{2} - i \frac{b-a}{2i}}{-\bar{\mathbb{L}}^*} \left| \begin{array}{c} \mathbb{L} \\ \frac{a+b}{2} + i \frac{b-a}{2i} \end{array} \right. \\ &- \left(\frac{a+b^2}{2} + \frac{b-a^2}{2i} + \mathbb{L}\bar{\mathbb{L}}^* \right) \frac{\frac{c+\bar{d}}{2} - i \frac{\bar{d}-\bar{c}}{2i}}{-\bar{\mathbb{L}}^*} \left| \begin{array}{c} \bar{\mathbb{L}} \\ \frac{c+\bar{d}}{2} + i \frac{\bar{d}-\bar{c}}{2i} \end{array} \right. \end{aligned}$$

$$z = x + iy$$

$$u = \alpha x + \beta y$$

$$v = \gamma x + \delta y$$

$$\frac{\alpha \mid \beta}{\gamma \mid \delta} = \frac{1}{1} \mid \frac{-1}{1} \frac{a \mid b}{c \mid d} = \frac{a+c}{b+d} \mid \frac{d-b}{a-c}$$

$$\underline{a+ib}\underline{x+iy} + \underline{c+id}\underline{x-iy} = \overbrace{a+cx + d-by}^{\substack{= \alpha}} + i \overbrace{b+dx + a-cy}^{\substack{= \beta}} = u + iv$$