

$$\begin{aligned} \mathbb{N} \triangleleft \mathfrak{h} &\ni {}_i \mathfrak{h} \text{ seq} \\ \mathbb{N} \triangleleft_{\infty} \mathfrak{h} &\supset \mathbb{N} \blacktriangleleft \mathfrak{h} \supset \mathbb{N} \triangleleft_{\omega} \mathfrak{h} \\ \left\{ \begin{array}{l} {}_i \mathfrak{h} \rightsquigarrow \\ {}_i \mathfrak{h} \rightsquigarrow \mathfrak{h} \end{array} \right. &\Rightarrow {}_i \mathfrak{h} \text{ bes} \end{aligned}$$

$$\bigvee_{m \in \mathbb{N}} \bigwedge_{i:j \geq m} {}_i \mathfrak{h} | {}_j \mathfrak{h} \leq 1 \Rightarrow \frac{{}_i \mathfrak{h}}{i \in \mathbb{N}} = \frac{{}_i \mathfrak{h}}{i \in m} \cup \frac{{}_i \mathfrak{h}}{i \geq m}$$

$$\bigvee_{m \in \mathbb{N}} \bigwedge_{n \geq m} {}_n \mathfrak{h} | \mathfrak{h} \leq 1 \Rightarrow \bigwedge_{n \in \mathbb{N}} {}_n \mathfrak{h} | \mathfrak{h} \leq 1 \vee \prod_m^n {}_m \mathfrak{h} | \mathfrak{h} < \infty$$

$$\mathbb{N} \blacktriangleleft \mathfrak{h} = \frac{{}_n \mathfrak{h}: {}_n \mathfrak{h} \rightsquigarrow}{\bigwedge_{\varepsilon > 0} \bigvee_{m \in \mathbb{N}} \bigwedge_{i:j \geq m} {}_i \mathfrak{h} | {}_j \mathfrak{h} \leq \varepsilon} \in \triangleleft_0^d \vee -1$$

$$\mathbb{N} \triangleleft_{\omega} \mathfrak{h} = \frac{{}_n \mathfrak{h}: {}_n \mathfrak{h} \rightsquigarrow \mathfrak{h}}{\bigvee_{\mathfrak{h} \in \mathfrak{h}} \bigwedge_{\varepsilon > 0} \bigvee_{m \in \mathbb{N}} \bigwedge_{n \geq m} {}_n \mathfrak{h} | \mathfrak{h} \leq \varepsilon}$$

$${}_n \mathfrak{h} \rightsquigarrow \mathfrak{h} \Rightarrow {}_n \mathfrak{h} \rightsquigarrow$$

$$i:j \geq \frac{{}_i \mathfrak{h}}{\varepsilon/2} \Rightarrow {}_i \mathfrak{h} | {}_j \mathfrak{h} \leq {}_i \mathfrak{h} | \mathfrak{h} + \mathfrak{h} | {}_j \mathfrak{h} \leq \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon$$

$$\mathfrak{h} \ni {}_n \mathfrak{h} \rightsquigarrow \left\{ \begin{array}{l} \mathfrak{h} \\ \mathfrak{h}' \end{array} \right. \xrightarrow[\text{eind}]{\lim} \mathfrak{h} = \mathfrak{h}' = \lim {}_n \mathfrak{h}$$

$$\mathfrak{h} \neq \mathfrak{h}' \xrightarrow{\text{asym}} \mathfrak{h} | \mathfrak{h}' > 0 \Rightarrow \bigvee_{\hat{m} \in \mathbb{N}} \bigwedge_{n \geq \hat{m}} {}_n \mathfrak{h} | \mathfrak{h}' < \frac{\mathfrak{h} | \mathfrak{h}'}{2}$$

$$\Rightarrow \mathfrak{h} | \mathfrak{h}' \leq \mathfrak{h} |_{m \vee \hat{m}} \mathfrak{h} + {}_m \vee \hat{m} \mathfrak{h} | \mathfrak{h}' < \mathfrak{h} | \mathfrak{h}' \quad \sharp$$

$${}_n \mathcal{H} | {}_{n+1} \mathcal{H} \leq 2^{-n} \Rightarrow {}_n \mathcal{H} \rightsquigarrow$$

$$\begin{aligned} \bigwedge_{\varepsilon > 0} \bigwedge_{m \in \mathbb{N}} 2^{-m} \leq \varepsilon &\Rightarrow \bigwedge_{q \geq p > m} {}_p \mathcal{H} | {}_q \mathcal{H} \leq {}_p \mathcal{H} | {}_{p+1} \mathcal{H} + \dots + {}_{q-1} \mathcal{H} | {}_q \mathcal{H} \\ &\leq 2^{-p} + 2^{-p-1} + \dots + 2^{1-q} \leq 2^{-p} \underbrace{1 + 2^{-1} + \dots}_{= 2^{1-p}} = 2^{1-p} \leq 2^{-m} \leq \varepsilon \end{aligned}$$

$\mathcal{H} | \mathcal{H}' = \lim {}_n \mathcal{H} | {}_n \mathcal{H}'$  halb-metric on  $\mathbb{N} \blacktriangleright \mathfrak{H}$

$$\mathfrak{H} \ni {}_n \mathcal{H}' \rightsquigarrow \Rightarrow \overline{{}_n \mathcal{H} | {}_n \mathcal{H}' - {}_m \mathcal{H} | {}_m \mathcal{H}'} \leq {}_n \mathcal{H} | {}_m \mathcal{H} + {}_n \mathcal{H}' | {}_m \mathcal{H}' \Rightarrow {}_n \mathcal{H} | {}_n \mathcal{H}' \rightsquigarrow$$

$$\mathbb{N} \blacktriangleleft \mathfrak{H} \in \triangleleft_0^d \text{ v-1}$$

$$\mathbb{N} \blacktriangleleft \mathfrak{H} \ni \mathfrak{H}^k \text{ Cauchy} \Rightarrow \bigwedge_{\varepsilon > 0} \bigvee_{\ell \geq 1/\varepsilon} \bigwedge_{i,j \geq \ell} \mathfrak{H}^i | \mathfrak{H}^j \leq \varepsilon$$

$$\bigwedge_{k \geq 1} \bigwedge_{n_k} \bigwedge_{p,q \geq n_k} \mathfrak{H}^k | \mathfrak{H}^k \leq \frac{1}{k}$$

$$\mathfrak{H}^k := \mathfrak{H}^k_{n_k} \in \mathfrak{H} \Rightarrow \bigwedge_{n \geq n_k} \mathfrak{H}^k | \mathfrak{H}^k \leq \frac{1}{k}$$

$$\bigwedge_{n \geq n_i \vee n_j} \mathfrak{H}^i | \mathfrak{H}^j \leq \mathfrak{H}^i | \mathfrak{H}^i + \mathfrak{H}^i | \mathfrak{H}^j + \mathfrak{H}^j | \mathfrak{H}^j \leq \frac{1}{i} + \mathfrak{H}^i | \mathfrak{H}^j + \frac{1}{j} \rightsquigarrow \frac{1}{i} + \mathfrak{H}^i | \mathfrak{H}^j + \frac{1}{j}$$

$$\Rightarrow \mathfrak{H}^i | \mathfrak{H}^j \leq \frac{1}{i} + \mathfrak{H}^i | \mathfrak{H}^j + \frac{1}{j} \Rightarrow \bigwedge_{i,j \geq \ell} \mathfrak{H}^i | \mathfrak{H}^j \leq \frac{1}{i} + \varepsilon + \frac{1}{j} \leq \frac{2}{\ell} + \varepsilon \leq 3\varepsilon \Rightarrow \mathfrak{H} \ni \mathfrak{H} \rightsquigarrow \text{Cau}$$

$$\mathfrak{H} \in \mathbb{N} \blacktriangleleft \mathfrak{H}$$

$$\bigwedge_{i \geq \ell} \bigwedge_{j \geq n_i \vee \ell} \mathfrak{H}^i | \mathfrak{H}^j \leq \mathfrak{H}^i | \mathfrak{H}^i + \mathfrak{H}^i | \mathfrak{H}^j \leq \frac{1}{i} + 3\varepsilon \leq \frac{1}{\ell} + 3\varepsilon \leq 4\varepsilon$$

$$\Rightarrow \mathfrak{H}^i | \mathfrak{H} = \lim_j \mathfrak{H}^i | \mathfrak{H}^j \leq 4\varepsilon \Rightarrow \mathfrak{H}^i | \mathfrak{H} \rightsquigarrow 0 \Rightarrow \mathfrak{H}^i \rightsquigarrow \mathfrak{H} \in \mathbb{N} \blacktriangleleft \mathfrak{H}$$