R ordered field

$$R \ni a_n \rightsquigarrow a \Leftrightarrow \bigwedge_{\varepsilon}^{>0} \bigvee_{m}^{\mathbb{N}} \bigwedge_{n}^{\geqslant m} \begin{cases} a - \varepsilon \leqslant a_n \leqslant a + \varepsilon \\ -\varepsilon \leqslant a_n - a \leqslant \varepsilon \\ \ulcorner a_n - a \urcorner \leqslant \varepsilon \end{cases}$$

$$\dot{a}_n \rightsquigarrow \dot{a} \overset{\text{sum}}{\underset{\text{rule}}{\Longrightarrow}} a_n + \acute{a}_n \rightsquigarrow a + \acute{a}$$

$$\bigwedge_{\varepsilon}^{>0} \bigvee_{m}^{\mathbb{N}} \bigwedge_{n}^{\geqslant m} -\varepsilon/2 \leqslant \dot{a}_n - \dot{a} \leqslant \varepsilon/2$$

$$-\varepsilon = -\varepsilon/2 - \varepsilon/2 \leqslant \underbrace{a_n - a} + \underbrace{\acute{a}_n - \acute{a}} = \underbrace{a_n + \acute{a}_n} - \underbrace{a + \acute{a}} \leqslant \varepsilon/2 + \varepsilon/2 = \varepsilon$$

$$\begin{cases} 0 \leqslant a_n \rightsquigarrow a & \Longrightarrow 0 \leqslant a \\ a \leftsquigarrow a_n \leqslant b_n \rightsquigarrow b & \Longrightarrow a \leqslant b \end{cases}$$

$$\dot{a}_n \rightsquigarrow \dot{a} \overset{\text{prod}}{\underset{\text{rule}}{\Longrightarrow}} a_n \cdot \acute{a}_n \rightsquigarrow a \cdot \acute{a}$$

$$\bigvee_{M}^{>0} \bigwedge_{n}^{\mathbb{N}} -M \leqslant \acute{a}_n \leqslant M$$

$$-\ulcorner a \urcorner \leqslant a \leqslant \ulcorner a \urcorner$$

$$\bigwedge_{\varepsilon}^{>0} \bigvee_{m}^{\mathbb{N}} \bigwedge_{n}^{\geqslant m} -\frac{\varepsilon}{M + \ulcorner a \urcorner} \leqslant \dot{a}_n - \dot{a} \leqslant \frac{\varepsilon}{M + \ulcorner a \urcorner}$$

$$-\varepsilon = -\frac{\varepsilon}{M + \ulcorner a \urcorner} M - \ulcorner a \urcorner \frac{\varepsilon}{M + \ulcorner a \urcorner} \leqslant \overbrace{a_n - a}\, \acute{a}_n + a\, \overbrace{\acute{a}_n - \acute{a}} = a_n\, \acute{a}_n - a\, \acute{a} \leqslant \frac{\varepsilon}{M + \ulcorner a \urcorner} M + \ulcorner a \urcorner \frac{\varepsilon}{M + \ulcorner a \urcorner} = \varepsilon$$

$$a_n \rightsquigarrow a \neq 0 \overset{\text{quot}}{\underset{\text{rule}}{\Longrightarrow}} \bigwedge_{n}^{\text{fast}} a_n \neq 0 \wedge \frac{1}{a_n} \rightsquigarrow \frac{1}{a}$$

$$\text{OE } a > 0 \Rightarrow \bigvee_{m_0}^{\mathbb{N}} \bigwedge_{n}^{\geqslant m_0} -\frac{a}{2} \leqslant a_n - a \leqslant \frac{a}{2} \Rightarrow \frac{a}{2} \leqslant a_n \Rightarrow \frac{a}{2a_n} \leqslant 1 \Rightarrow -1 \leqslant -\frac{a}{2a_n}$$

$$\bigwedge_{\varepsilon}^{>0} \bigvee_{m}^{\geqslant m_0} \bigwedge_{n}^{\geqslant m} -\frac{\varepsilon a^2}{2} \leqslant a_n - a \leqslant \frac{\varepsilon a^2}{2}$$

$$-\varepsilon \leqslant -\varepsilon \frac{a}{2a_n} = -\frac{\varepsilon a^2}{2} \frac{1}{a_n a} \leqslant \frac{a_n - a}{a_n a} = \frac{1}{a} - \frac{1}{a_n} \leqslant \frac{\varepsilon a^2}{2} \frac{1}{a_n a} = \varepsilon \frac{a}{2a_n} \leqslant \varepsilon$$

$$\mathbb{R} \ni a_n \text{ bes}$$

$$\text{upper envelope } \bar{\text{ⵅ}}^m = \curlyvee \frac{a_n}{m \leqslant n} = \curlyvee_{m \leqslant n} a_n = \curlyvee \text{ⵅ}^{\geqslant m}$$

$$\text{lower envelope } \underline{\text{ⵅ}}^m = \curlywedge \frac{a_n}{m \leqslant n} = \curlywedge_{m \leqslant n} a_n = \curlywedge \text{ⵅ}^{\geqslant m}$$

$$\underline{a}_\infty = \liminf a_n \nwarrow \underline{\text{ⵅ}}^m \leqslant \text{ⵅ}^m \leqslant \bar{\text{ⵅ}}^m \searrow \limsup a_n = \bar{a}_\infty$$

$$0/ \qquad \underline{a}_\infty \leqslant \bar{a}_\infty$$

$$a_n \rightsquigarrow \text{ⵅ}^\infty \Rightarrow \underline{a}_\infty = \text{ⵅ}^\infty = \bar{a}_\infty$$

$$\bigwedge_{\varepsilon > 0} \bigvee_{n_0} \bigwedge_{n \geqslant n_0} \underbrace{\text{ⵅ}^\infty - \varepsilon}_{\text{lower bd}} \leqslant a_n \leqslant \underbrace{\text{ⵅ}^\infty + \varepsilon}_{\text{upper bd}} \Rightarrow \text{ⵅ}^\infty - \varepsilon \leqslant \underset{\text{glb}}{\underline{a}_n} \leqslant \underset{\text{lub}}{\bar{a}_n} \leqslant \text{ⵅ}^\infty + \varepsilon$$

$$\begin{cases} \bar a_\infty \\ \underline{a}_\infty \end{cases} \in \ \mathrm{LIM}\ a_n$$

$$\bigwedge_{m>0} \bar\lambda^m - \frac{1}{m} \text{ keine obere Schranke von } \lambda^{\geqslant m} \Rightarrow \bigvee_{j(m)\geqslant m} \bar\lambda^m - \frac{1}{m} < \lambda^{j(m)}$$

$$\bigwedge_{\varepsilon>0} \bigvee_{m_0 \geqslant \frac{1}{\varepsilon}} \bigwedge_{m \geqslant m_0} \bar a \leqslant \bar\lambda^m \leqslant \bar a + \varepsilon$$

$$\Rightarrow \bar a_\infty - \varepsilon \leqslant \bar a_\infty - \frac{1}{m} \leqslant \bar\lambda^m - \frac{1}{m} < \lambda^{j(m)} \underset{j(m)\geqslant m}{\leqslant} \bar\lambda^m \leqslant \bar a_\infty + \varepsilon \Rightarrow \lambda^{j(m)} \rightsquigarrow \bar a_\infty \in \ \mathrm{LIM}\ a_n$$

$$b \in \ \mathrm{LIM}\ a_n \Rightarrow \underline{a}_\infty \leqslant b \leqslant \bar a_\infty$$

$$\bigvee_{j(m)\geqslant m} \lambda^{j(m)} \rightsquigarrow b$$

$$\lightning\ b > \bar a_\infty \underset{\varepsilon = \left(b - \bar a_\infty\right)/2}{\Longrightarrow} \bigvee_{m_0} \bigwedge_{m\geqslant m_0} \frac{\bar a_\infty + b}{2} = b - \varepsilon \leqslant \lambda^{j(m)} \leqslant \bar\lambda^m \Rightarrow \bar a_\infty = \lim \bar\lambda^m \geqslant \frac{\bar a_\infty + b}{2} > \bar a_\infty\ \lightning$$

$$\begin{cases} \limsup a_n = \max\ \mathrm{LIM}\ a_n & \text{groesster Haeufungspunkt} \\ \liminf a_n = \min\ \mathrm{LIM}\ a_n & \text{kleinster Haeufungspunkt} \end{cases}$$