$$\begin{cases} {}^{x:y}P dx + {}^{x:y}Q dy = 0 \\ P_y \neq Q_x \end{cases}$$

$${}^{x:y}F \text{ int factor } \Leftrightarrow {}^{x:y}(FP)\,dx + {}^{x:y}(FQ)\,dy = 0 \text{ int } \Leftrightarrow (FP)_y = (FQ)_x \Rightarrow F = \frac{F_y P - F_x Q}{Q_x - P_y}$$

$$\underbrace{2y + xy}\,dx + 2x dy = 0 \Longrightarrow \frac{P_y - Q_x}{Q} = \frac{1}{2} \underset{\text{int fact}}{\Longrightarrow} {}^{x}F = e^{x/2}$$

$$\Longrightarrow \int\limits_{dx} e^{x/2}\,(2y + xy) + 2 \int\limits_{dy} x\,e^{x/2} = \int\limits_{dxdy} e^{x/2}\,(x+2) \underset{x}{\Longrightarrow} y\,e^{x/2} = C \underset{y\,(3) = \sqrt{2}}{\Longrightarrow} C = \sqrt{72e^3}$$

$$\underbrace{3xy + y^2}\,dx + \underbrace{x^2 + xy}\,dy = 0 \Longrightarrow \frac{P_y - Q_x}{Q} = \frac{1}{x} \underset{\text{int fact}}{\Longrightarrow} {}^{x}F = x \Longrightarrow x^3 y + \frac{x^2 y^2}{2} = C$$

$$xdy - ydx = 2\,x^2\,y^2\,dy \Longrightarrow \frac{P_y - Q_x}{Q} = -\frac{2}{x} \underset{\text{int fact}}{\Longrightarrow} {}^{x}F = x^{-2}$$

$$ydx + \underbrace{2x - ye^y}\,dy = 0 \Longrightarrow \frac{Q_x - P_y}{P} = \frac{1}{y} \underset{\text{int fact}}{\Longrightarrow} G_y = y \Longrightarrow$$

$$\underbrace{\int\limits_{dx} y^2}_{= xy^2} + \underbrace{\int\limits_{dy} \left(2xy - y^2 e^y\right)}_{= xy^2 - \int\limits_{dy} y^2 e^y = xy^2 - \left(y^2 e^y - 2\int\limits_{dy} y e^y\right)} = \int\limits_{dxdy} 2y = x\,y^2 + C \Longrightarrow y^2\,x + e^y\,\left(2y - y^2 - 2\right) = C$$

$$xdy + \underbrace{2y - xe^x}\,dx = 0 \Longrightarrow y = \frac{C - e^x\left(2x - x^2 - 2\right)}{x^2}$$

$$\underbrace{x + 2}\,{}^{y}\mathfrak{s}\,dx + x\,{}^{y}\mathfrak{c}\,dy = 0 \Longrightarrow \frac{P_y - Q_x}{Q} = \frac{x+1}{x} \underset{\text{int fact}}{\Longrightarrow} {}^{x}F = x\,e^x$$