$$\frac{dy}{dx} = {}^{x}p \, {}^{y}q$$

$$\text{const solution } {}^{y}q = 0 = \frac{dy}{dx}$$

$$\text{non-const solution } {}^{y}q \neq 0 \Longrightarrow \frac{dy}{{}^{y}q} = dx \, {}^{x}p \Longrightarrow \int \frac{dy}{{}^{y}q} = \int dx \, {}^{x}p + C$$

$$\Longrightarrow \text{ solve for } y \Longrightarrow \text{ find C by initial cond}$$

$$\frac{dy}{dx} = \alpha y + \beta = \alpha \left(y + \beta/\alpha\right)$$

$$\frac{dy}{dx} + 4y = 0$$

$$\frac{dy}{y + \beta/\alpha} = \alpha dx \Longrightarrow \log\left(y + \beta/\alpha\right) = \alpha x + C \Longrightarrow y + \beta/\alpha = \mathfrak{e}^{\alpha x} \, \mathfrak{e}^{C} \Longrightarrow y = \mathfrak{e}^{\alpha x} \, \mathfrak{e}^{C} - \beta/\alpha$$

$${}^{a}y = b \Longrightarrow b = \mathfrak{e}^{\alpha a} \, \mathfrak{e}^{C} - \beta/\alpha \Longrightarrow \mathfrak{e}^{\alpha a} \, \mathfrak{e}^{C} = b + \beta/\alpha \Longrightarrow \alpha a + C = \log\left(b + \beta/\alpha\right)$$

$$\frac{dy}{dx} = kxy \Longrightarrow \overline{y} = \underset{>0}{A} \exp\left(kx^2/2\right) \Longrightarrow y = \underset{\in \mathbb{R}}{B} \exp\left(kx^2/2\right)$$

$$\frac{dy}{dx} = y \ln y$$

$$\frac{dy}{dx} = \frac{3x^n}{y^n} \Longrightarrow y = +\widehat{3x^2 + C}^{\,1/(n+1)} \underset{y(0) = 4}{\Longrightarrow} C = 4^{n+1}$$

$$x\frac{dy}{dx} = y+1 \Longrightarrow \left|y+1\right| = \underset{>0}{A}\left|x\right|$$

$$x\frac{dy}{dx} = 2\sqrt{y-1} \underset{x\neq 0}{\Longrightarrow} y = 1 + \left(\ln\left|x\right| + C\right)^2$$

$$\frac{dy}{dx} = \left(2x+1\right)e^{-y} \underset{y(0)=1}{\Longrightarrow} y = \ln\left(x^2+x+e\right)$$

$$\frac{dy}{dx} = \frac{y}{x\ln x} \underset{y(3)=4}{\Longrightarrow} y = 4\frac{\ln\left|x\right|}{\ln 3}$$

$$\frac{dy}{dx} = x^3\,y^2 + y^2 = \left(x^3+1\right)y^2 \Longrightarrow \begin{cases} y = -\left(\dfrac{x^4}{4} + x + C\right)^{-1} & \text{non-const} \\ y = 0 & \text{const} \end{cases}$$

$$\frac{dy}{dx} = \frac{xe^x}{y} \Longrightarrow y = -\sqrt{2e^x\left(x-1\right) + C} \underset{y(0)=-5}{\Longrightarrow} C = 27$$

$$\frac{dy}{dx} = \sec y \Longrightarrow \sin y = \int \frac{dy}{\sec y} = x + C \underset{y(0)=0}{\Longrightarrow} C = 0 \Longrightarrow y = {}^{x}\not{\mathfrak{S}}$$

$$\underbrace{y^2 + 1}\, dy = 2xydx$$