

$$\frac{dy}{dx} + {}^x p y = {}^x q y^\alpha: \quad \alpha \neq 0:1$$

$${}^x z = {}^x y^{1-\alpha} \Rightarrow \frac{dz}{dx} + (1-\alpha) {}^x p {}^x z = (1-\alpha) {}^x q \text{ lin}$$

$$\frac{dz}{dx} = (1-\alpha) y^{-\alpha} \frac{dy}{dx} = (1-\alpha) y^{-\alpha} ({}^x q y^\alpha - {}^x p y) = (1-\alpha) ({}^x q - {}^x p z)$$

$$\begin{cases} \underline{y} + y = y^2 \\ y(0) = 4 \end{cases} \Rightarrow y = \sqrt{1 + C e^x}^{-1} \underset{y(0)=4}{\Rightarrow} C = -3/4$$

$$\begin{cases} \underline{y} + \frac{y}{x} = xy^2 \\ y(1) = 1/2 \end{cases} \Rightarrow y = \sqrt{3x - x^2}^{-1}$$

$$\alpha = 2: \quad z = \frac{1}{y} \Rightarrow \underline{z} - \frac{z}{x} = -x \Rightarrow z = x \int \frac{-x}{x} dx = -x^2 + Cx \underset{z(1)=2}{\Rightarrow} C = 3$$

$$x^2 dy + (y^2 - xy) dx = 0 \Rightarrow$$

$$\underline{y} - \frac{y}{x} = -\frac{y^2}{x^2} \Rightarrow z = \frac{1}{y} \Rightarrow \underline{z} + \frac{z}{x} = \frac{1}{x^2}$$

$$\begin{cases} 3\underline{y} + y = \underline{1-2x} y^4 \\ y(0) = 1/2 \end{cases} \Rightarrow y = (9e^x - 2x - 1)^{-1/3}$$

$$\underline{y} + \frac{y}{3} = \frac{1-2x}{3} y^4 \Rightarrow z = \frac{1}{y^3} \Rightarrow \underline{z} - z = 2x - 1 \Rightarrow z = e^x \int \frac{2x-1}{e^x} dx = -2x - 1 + C e^x \underset{z(0)=8}{\Rightarrow} C = 9$$

$$\begin{cases} \underline{y} - y^2 e^{2x} = -y \\ y(0) = 0 \end{cases}$$

$$\begin{cases} xy\underline{y} - y^2 = x \\ y(1) = 1 \end{cases}$$