$$Ly = a\,\underline{\underline{y}} + 2b\,\underline{y} + cy = 0: \quad \ddot{x} + 2\mu\,\dot{x} + \omega_0^2 x = 0$$

$$\text{Ansatz } {}^{x}y_{\lambda} = {}^{\lambda x}\mathfrak{e}$$

$$\overbrace{{}^{x}Ly_{\lambda}} = {}^{\lambda x}\mathfrak{e}\,\underbrace{a\lambda^2 + 2b\lambda + c}$$

$$\text{char equ } a\,\lambda^2 + 2b\lambda + c = 0 = \lambda^2 + 2\mu\lambda + \omega_0^2 \Leftrightarrow \lambda_{0:1} = -\frac{b}{a} \pm \frac{\sqrt{b^2 - ac}}{a} = -\mu \pm \sqrt{\mu^2 - \omega_0^2}$$

$$\text{overdamping } b^2 > ac: \quad \mu > \omega_0 \Longrightarrow \lambda_0 \neq \lambda_1 \in \mathbb{R}: \quad \lambda_{0,1} = -\mu \pm \sqrt{\mu^2 - \omega_0^2} < 0 \Longrightarrow \begin{cases} {}^{x}\mathbb{1}^{0} = {}^{\lambda_0 x}\mathfrak{e} \searrow \\ {}^{x}\mathbb{1}^{1} = {}^{\lambda_1 x}\mathfrak{e} \searrow \end{cases}$$

$$\text{underdamping } b^2 < ac: \quad 0 \leqslant \mu < \omega_0 \Longrightarrow \lambda_{0:1} = p \pm iq = -\mu \pm i\sqrt{\omega_0^2 - \mu^2}$$

$$\Longrightarrow \begin{cases} {}^{x}\mathbb{1}^{0} = {}^{(p+iq)\,x}\mathfrak{e} = {}^{px}\mathfrak{e}\left({}^{qx}\mathfrak{c} + i\,{}^{qx}\mathfrak{s}\right) \\ {}^{x}\mathbb{1}^{1} = {}^{(p-iq)\,x}\mathfrak{e} = {}^{px}\mathfrak{e}\left({}^{qx}\mathfrak{c} - i\,{}^{qx}\mathfrak{s}\right) \end{cases} \quad \Longrightarrow \begin{cases} {}^{x}_{\mathbb{R}}\mathbb{1}^{0} = {}^{px}\mathfrak{e}\,{}^{qx}\mathfrak{c} = {}^{-\mu t}\mathfrak{e}\cos x\sqrt{\omega_0^2 - \mu^2} \\ {}^{x}_{\mathbb{R}}\mathbb{1}^{1} = {}^{px}\mathfrak{e}\,{}^{qx}\mathfrak{s} = {}^{-\mu t}\mathfrak{e}\sin x\sqrt{\omega_0^2 - \mu^2} \end{cases}$$

$$\text{no damping } \mu = 0: \quad \begin{cases} {}^{x}\mathbb{1}^{0} = \cos\omega_0 x \\ {}^{x}\mathbb{1}^{1} = \sin\omega_0 x \end{cases}$$

$$\text{critical damping } b^2 = ac: \quad \mu = \omega_0 \Longrightarrow \lambda_{0,1} = -\frac{b}{a} = -\mu \text{ double } \Longrightarrow \begin{cases} {}^{x}\mathbb{1}^{0} = \mathfrak{e}^{-bx/a} = {}^{-\mu x}\mathfrak{e} \\ {}^{x}\mathbb{1}^{1} = x\mathfrak{e}^{-bx/a} = x^{-\mu x}\mathfrak{e} \end{cases}$$

$$L\left.\frac{dy_\lambda}{d\lambda}\right|_{\lambda=\lambda_1} = \frac{d}{d\lambda}L\,\curlywedge_{\lambda} = \frac{d}{d\lambda}\,{}^{\lambda x}\mathfrak{e}\,\underbrace{a\lambda^2 + 2b\lambda + c} = \underbrace{\frac{d}{d\lambda}\,{}^{\lambda x}\mathfrak{e}}\,\underbrace{a\lambda^2 + 2b\lambda + c} + {}^{\lambda x}\mathfrak{e}\,\frac{d}{d\lambda}\underbrace{a\lambda^2 + 2b\lambda + c}$$

$$= x\,{}^{\lambda x}\mathfrak{e}\,\underbrace{a\lambda^2 + 2b\lambda + c} + {}^{\lambda x}\mathfrak{e}\,\frac{d}{d\lambda}\underbrace{a\lambda^2 + 2b\lambda + c} \underset{\lambda=\lambda_1}{=} 0$$

$$\underline{\underline{y}} + 4\,\underline{y} + 3y = 0 \begin{cases} {}^{0}y = 0 \\ {}^{0}\underline{y} = 1 \end{cases}$$

$$\underline{\underline{y}} + 4\,\underline{y} + 5y = 0 \begin{cases} {}^{0}y = 1 \\ {}^{\pi/2}y = -1 \end{cases}$$

$$\underline{\underline{y}} - 2\,\underline{y} + y = 0 \begin{cases} {}^{0}y = -2 \\ {}^{0}\underline{y} = 4 \end{cases}$$

$$\underline{\underline{y}} - \underline{y} + \frac{y}{2} = 0 \begin{cases} {}^{0}y = -1 \\ {}^{\pi}y = 1 \end{cases} \quad y = e^{x/2}\left(c_1 \cos\frac{x}{2} + c_2 \sin\frac{x}{2}\right): \quad c_1 = -1: \quad c_2 = e^{-\pi/2}$$

$$3\underline{\underline{y}} - 8\underline{y} - 3y = 0 \begin{cases} {}^{-3}y = e \\ {}^{3}y = e \end{cases}$$

$$\underline{\underline{y}} - 7\underline{y} + 12y = 0 \begin{cases} {}^{0}y = 2 \\ {}^{0}\underline{y} = 7 \end{cases}$$

$$\underline{\underline{y}} + 6\underline{y} + 9y = 0 \begin{cases} {}^{0}y = 1 \\ {}^{0}\underline{y} = 0 \end{cases}$$

$$\underline{\underline{y}} + 4y = 0 \begin{cases} {}^{0}y = 1 \\ {}^{0}\underline{y} = 0 \end{cases}$$

$$\underline{\underline{y}} + 4\underline{y} + 4y = 0 \begin{cases} {}^{0}y = 1 \\ {}^{0}\underline{y} = 0 \end{cases}$$

$$\underline{\underline{y}} + 6\underline{y} + 13y = 0 \begin{cases} {}^{0}y = \\ {}^{0}\underline{y} = \end{cases} \quad {}^{x}1 = e^{-3x}\left(c_1 \cos 2x + c_2 \sin 2x\right)$$

$$\underline{\underline{y}} + 2\underline{y} + y = 0 \begin{cases} {}^{1}y = 0 \\ {}^{0}\underline{y} = 4 \end{cases} \quad {}^{x}1 = 2e^{-x}(x-1)$$