$$\overline{\overline{d\mathbb{U}}} = \overline{\frac{d\mathbb{U}}{dt}}\,\overline{dt} \text{ arc length measure}$$

$$\overline{\overline{\frac{d\mathbb{U}}{dt}}} = \overbrace{\frac{d\mathbb{U}^{2}{}_{1}}{dt} + \cdots + \frac{d\mathbb{U}^{2}{}_{n}}{dt}}^{1/2} \text{ speed}$$

$$\int_{dx}^{C} {}^{x}\mathbb{1} := \int_{dt}^{\mathbb{I}} {}^{t}\mathbb{U}\mathbb{1}\,\overline{\overline{\frac{d\mathbb{U}}{dt}}} = \int_{dt}^{\mathbb{I}} {}^{t\mathbb{U}^{1}:\cdots:t\mathbb{U}^{n}}\mathbb{1}\,\overbrace{\frac{d\mathbb{U}^{2}{}_{1}}{dt} + \cdots + \frac{d\mathbb{U}^{2}{}_{n}}{dt}}^{1/2}$$

$$\text{arc length } \ell(C) = \int_{dx}^{C} 1 = \int_{dt}^{\mathbb{I}} \overline{\overline{\frac{d\mathbb{U}}{dt}}}$$

$$\overline{\overline{dR}} = \overline{\overline{\frac{dR}{dt}}}\,\overline{dt} \text{ arc length measure}$$

$$\overline{\overline{\frac{dR}{dt}}} = \overbrace{\frac{dx}{dt}^{2} + \frac{dy}{dt}^{2} + \frac{dz}{dt}^{2}}^{1/2} \text{ speed}$$

$$\int^{C} {}^{R}\mathbb{1} := \int_{dt}^{\mathbb{I}} {}^{R(t)}\mathbb{1}\,\overline{\overline{\frac{dR}{dt}}}$$

$$= \int_{dt}^{\mathbb{I}} {}^{t_x:t_y:t_z}\mathbb{1}\,\overbrace{\frac{dx}{dt}^{2} + \frac{dy}{dt}^{2} + \frac{dz}{dt}^{2}}^{1/2}$$

$$\text{arc length } \ell(C) = \int^{C} 1 = \int_{dt}^{\mathbb{I}} \overline{\overline{\frac{dR}{dt}}}$$