

$$\mathbb{C} \cup \infty = \frac{z}{\begin{array}{c} w \\ z \in \mathbb{C} \ni w: \quad z \neq 0 \vee w \neq 0 \end{array}}$$

$$(S) = \frac{z \in \bar{\mathbb{C}}}{\begin{bmatrix} 1 & z^* \end{bmatrix} \frac{d}{b} \begin{array}{c|c} b^* & 1 \\ \hline a & z \end{array} = az^*z + bz^* + b^*z + d = 0}$$

$$\text{circle } r^2 = \overline{z+b}^2 = zz^* + b^*z + bz^* + b^*b \Rightarrow a = 1$$

$$d = b^*b - r^2 \Rightarrow b^*b - ad = r^2 > 0$$

$$\text{line } Ax + By + d = 0 \Rightarrow b = \frac{A + iB}{2} \wedge z b^* + z^* b + d = 0 \wedge a = 0 \Rightarrow b^*b - ad = \frac{A^2 + B^2}{4} > 0$$

$$\text{converse } az^*z + bz^* + b^*z + d = 0 \Rightarrow \begin{cases} A = b + b^* \wedge B = (b - b^*)/i & a = 0 \\ o = -b/a \wedge r^2 = o^2 - d/a = \frac{b^*b - ad}{a^2} > 0 & a \neq 0 \end{cases}$$

$$\bar{\mathbb{R}} = \frac{0}{-i} \begin{array}{c|c} & i \\ \hline & 0 \end{array}$$

$$i\bar{\mathbb{R}} = \frac{0}{-1} \begin{array}{c|c} & -1 \\ \hline & 0 \end{array}$$

$$\mathbb{T} = \frac{-1}{0} \begin{array}{c|c} & 0 \\ \hline & 1 \end{array}$$